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# Effects of recycled PET fibres on the mechanical properties and seawater curing of Portland cement-based concretes



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# HIGHLIGHTS

• We present an experimental study on the mechanical properties and seawater curing of RPETFRC.

• We examine compressive strength, first crack strength, and energy absorption capacity of RPETFRC.

• Special attention is given to the influence of the mix-design on RPETFRC properties.

• The presented results show that seawater conditioning significantly lowers the ultimate ductility of the analysed RPETFRC.

• The same conditioning instead leads to minor modifications of the compressive strength and first-crack strength.

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# ABSTRACT

This paper deals with an experimental study on the mechanical properties of recycled polyethylene terephthalate fibre-reinforced concrete (RPETFRC) and its durability in an aggressive seawater environment. A Portland limestone cement-based concrete with a 0.38 water/cement ratio is used to cast cubic and prismatic specimens, in association with two different PET fibres obtained through extrusion of recycled PET flakes (R-PET). Some of these specimens were conditioned in the Salerno harbour seawater for a period of 6/12 months. Compressive strength and four-point bending tests are performed in order to investigate the mechanical properties of such RPETFRCs. Comparison of the present results and those in the literature for air-cured RPETRCs highlights the influence of the analysed R-PET fibres on the mechanical properties of concretes showing different water/cement ratios and binders. The given results for seawater-cured specimens demonstrate that such a curing condition slightly modifies the first-crack strength and markedly reduces the toughness of the RPETFRCs examined in the present work.

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1. Introduction

The attention towards an effective recycling of post-consumer plastics in different industrial sectors has grown considerably over the past two decades. The interest in plastic waste materials mainly originates from environmental reasons, due to the fact that post-consumer plastics are the most relevant wastes with a low rate of biodegradation, and in consideration of the severe environmental problems created by the disposal of such materials in landfills, or their floating in the ocean. On the other hand, in recent years, it has been shown that recycled plastic can be profitably used to manufacture low-cost aggregates and/or reinforcing fibres of cementitious materials in the construction industry (refer, e.g., to [1–10] and therein references). It is well known that plain cementitious mixes may suffer considerable shrinkage during curing and can be affected by diffuse cracking. The addition of a suitable amount of reinforcing fibres to the mix design can effectively contrast such phenomena, leading to significant increases in the material toughness and durability [11-13]. Nowadays, fibre-reinforced concretes (FRCs) are widely employed for industrial floors, tunnel coatings, and the retrofitting of structures exposed to chemical attacks or undergoing structural rehabilitation. Natural (cellulose, carbon, cotton, coconut, agave, jute, etc.) or synthetic (steel, polypropylene, polyethylene, polyester, nylon, Kevlar, etc.) reinforcing fibres are frequently used. It has been recognized that plastic fibres offer several advantages over more traditional steel fibres, including: markedly lower weight for equal volume content; lower transportation costs; higher corrosion resistance, major impermeability of the fibre-reinforced concrete; enhanced compatibility with additives; lower thermal conductivity; higher workability;

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lower wear and tear of machinery and equipment for material laying.

The extrusion of plastic filaments from flakes of recycled polyethylene terephthalate (R-PET) has received particular attention in the technical literature, since it has been shown that R-PET fibres can conveniently replace virgin plastic fibres in a variety of engineering applications [1–9]. The construction industry makes use of R-PET recovered from post-consumer bottles in the form of structural and non-structural lightweight aggregates [10], as well as reinforcing fibres (or strips) for eco-friendly concretes [1–7] or mortars [8,9]. There is a general agreement in the literature about the beneficial effects of R-PET fibres on the first-crack strength and/or the fracture toughness of R-PET fibre-reinforced concretes (RPETFRCs) and mortars [1-9]. More puzzling is the response of RPETFRCs in terms of compressive strength, since some studies report slight increases (increases equal to 8.4%, 13.8% and 6.0% for 1% fibre volume fraction and water/cement ratios equal to 65%. 60% and 55%, respectively, cf. [2]), or slight decreases (-7% for 1% fibre volume fraction and a water/cement ratio equal to 0.41, cf. [3]) of such a quantity, over unreinforced concrete (UNRC), while Fraternali et al. [1] highlight marked increments of the same property (up to 22-35%) for some special RPEFTRFCs (1% fibre volume fraction; water/cement ratio equal to 53%), which are made up with a pozzolana cement (cement class CEM IV/B 32.5 R according to the European standard UNI EN 197-1 [14]) and industrially manufactured R-PET fibres. The above results indicate that the mechanical response of RPETFRCs is markedly influenced by fibre and mix designs, and is particularly sensitive to the water/cement ratio [1-3].

The durability of cementitious materials, due to aging and/or aggressive environments, is extremely topical and of great technical interest [15–26]. Special importance is given to the degradation induced by material exposure to seawater, which is extremely important in the case of submarine structures and constructions in coastal areas, such as a large number of the constructions lying in the Province of Salerno. The mechanism of concrete deterioration due to seawater is well established in scientific literature. and refers to three different exposure zones: submerged, splash and atmospheric [15–17]. The submerged zone is continuously covered by seawater; the splash zone is subject to continuous wetting and drying; and the atmospheric zone is above the splash zone and subject to occasional seawater spray. Concrete in the submerged zone is less vulnerable than concrete lying in the other two zones, since prolonged immersion usually provides sufficiently stable temperature and moisture conditions, which prevent deteriorations due to freeze-thaw and/or wet-dry cycles, and differential volume changes due to moisture gradients between the surface and the interior of the structure. On the other hand, submerged concrete may suffer from strength and/or material loss resulting from the reaction of sulphates, chlorides and/or  $\rm Mg^{2+}$  ions with the cement paste [17-19]. Additional degradation mechanisms, due to seawater exposure, may arise from chemical attacks on the reinforcing fibres. The corrosion of steel fibres in concrete has been extensively investigated over the last 25 years, due to its influence on concrete spalling; shrinking of the fibre crosssection area; and reduction of the fibre-concrete bonding capacity [20-24]. An extensive study on the durability of concretes reinforced with different fibres (polypropylene/PP, polyvinyl-alcohol/ PVA, hooked-end steel/St, and cellulose/Cell) has been recently carried out by a group of researchers from the University of Florida, on considering prolonged exposure (27 months) to simulated saltwater (immersed and wet/dry) and swamp (acid) environments [25]. As pointed out by Won et al. in [26], limited results are available in the literature about the long-term performance of FRCs equipped with R-PET fibres. The experimental study presented by Won et al. [26] deals with the RPETFRC exposure to alkaline, saline, CaCl<sub>2</sub>, sulphuric acid and sodium sulphate environments, showing that such a composite material may be particularly sensitive to alkali and sulphuric acid attacks.

The present work is an extension of a previous experimental study by Fraternali et al. [1] on the thermo-mechanical properties of concretes reinforced with R-PET fibres. A mix design based on a Portland limestone cement (cement class CEM II/A-LL 32.5 R according to UNI EN 197-1 [14]) and two different kinds of R-PET fibres are analysed. The R-PET fibres of the first type have a straight profile and high tensile strength (PET/a), while those of the second type have a crimped profile and relatively low tensile strength (PET/c). We examine both air-cured and seawater-cured specimens. After unmoulding, the first ones were air-cured for 28 days, and subsequently tested, in the Structural Engineering Laboratory of the University of Salerno, adopting a curing temperature of  $20 \pm 5$  °C in a room at high relative humidity ("Lab" specimens). The seawater-cured specimens were instead submerged in a reserved area of the port of Salerno for periods of 6 and 12 months ("Sea" specimens), after the initial period of 28 days air-curing. We focus our attention on the compressive strength, first-crack strength, energy absorption capacity and ductility indices of the examined RPETFRCs.

A first objective of this study is to characterize the effects of the binder nature, water/cement ratio and R-PET fibre properties on the mechanical response of air-cured RPETFRCs by establishing extensive comparisons between the present results and those in the literature on the same topics [1-3,25]. A second goal is the characterization of RPETFRC durability under prolonged seawater curing, with the aim of obtaining useful information on the resistance of this material to a saline environment and chemical attacks from ions such as chloride, sodium, potassium, magnesium, calcium and sulphate.

### 2. Specimen preparation

# 2.1. Recycled PET fibres

The plastic fibres used in this study were produced by mean of a R-PET flake extrusion lines [1]. We examine the fibres labelled as PET/a and PET/c in the reference study [1], whose properties are listed in Table 1. PET/a fibres have a straight profile and exhibit 550 MPa tensile strength, while PET/c fibres are crimped and feature 274 MPa tensile strength (cf. also Fig. 1 in [1]).

#### 2.2. Concrete test specimens

Several unreinforced (plain) concrete (UNRC) and R-PET fibrereinforced concrete (RPETFRC) specimens were prepared to study the influence of the R-PET reinforcement on the mechanical properties of the final material. Cubic (150 mm  $\times$  150 mm  $\times$  150 mm) and prismatic (150 mm  $\times$  150 mm  $\times$  600 mm) concrete specimens were prepared using the mix design shown in Table 2, which

Table 1

Geometrical and n	nechanical	properties of	of employed	R-PET	fibres.
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Type of fibre	Specific gravity (kg/m <sup>3</sup> )	Profile	Diameter (mm)	Length (mm)	Tensile strength (MPa)	Ultimate strain (%)
PET/a	1340	Smooth	1.10	40	550	27
PET/c	1340	Crimped	0.70	52	274	19



Fig. 1. (a) submersion of concrete samples in the Port of Salerno; (b) recovery of the specimens after curing.

#### Table 2 Concrete mix design.

Concrete type	Coarse aggregate (10–20 mm) (kg/m <sup>3</sup> )	Medium aggregate (4–10 mm) (kg/m <sup>3</sup> )	Sand (0–4 mm) (kg/m <sup>3</sup> )	CEM II A-LL 32.5 (kg/m <sup>3</sup> )	Fluidizing additive SKY 624 (kg/m <sup>3</sup> )	Water (Lt./m <sup>3</sup> )	Water/cement ratio	Fibres (kg/m <sup>3</sup> )
UNRC	605.0	170.0	944.1	496.0	4,35	187.9	0.38	-
RPETFRC/a	605.0	170.0	944.1	496.0	4,35	187.9	0.38	13.4
RPETFRC/c	605.0	170.0	944.1	496.0	4,35	187.9	0.38	13.4

makes use of the Portland limestone cement named CEM II/A-LL 32.5 R in the European standard UNI EN 197-1 [14]. This mix design aims to manufacture a concrete featuring C30/37 compressive strength class according to UNI EN 206-1 [27].

It is worth noting that such a concrete has a considerably smaller water/cement ratio (0.38 vs. 0.53), and greater compressive strength class (C30/37 vs. C25/30), than the pozzolana cementbased concrete analysed by Fraternali et al. [1]. Hereafter, we name UNRC the concrete specimens using the mix design in Table 2 and no reinforcing fibres; RPETFRC/a the concrete specimens based on the same mix design and reinforced with PET/a fibres at 1% volume content; and RPETFRC/c the concrete specimens based on the mix design in Table 2 and reinforced with PET/c fibres at 1% volume content. The latter were obtained by adding 13.4 gr of PET fibres per litre of concrete (cf. Table 1) and combining the mix by means of a concrete mixer.

All the specimens were unmoulded 3 days after casting and initially air-cured for 28 days at room temperature  $(20 \pm 5 \text{ °C})$  and in high relative humidity conditions, in a dedicated room of the Structural Engineering Laboratory of the University of Salerno. The specimens were successively separated into the following three groups:

Table 3Chemical composition of real or simulated seawaters.

Ions	Salerno port (g/litre)	Normal seawater [15] (g/litre)	Simulated seawater [25] (g/litre)
K*	0.36	0.40	0.18
Na <sup>+</sup>	10.23	11.00	31.56
Ca <sup>2+</sup>	0.39	0.43	0.18
Mg <sup>2+</sup>	1.26	1.33	2.48
Cl <sup>-</sup>	16.84	19.80	50.06
$SO_4^{2-}$	0.86	2.76	9.66
Salt content (by weight)	2.99	3.50	11.50

- Lab specimens, to be tested without any additional curing.
- Sea/6 specimens, to be tested after 6 months of seawater submersion.
- Sea/12 specimens, to be tested after 12 months of seawater submersion.

The seawater conditioning was carried out in a reserved area of the Port of Salerno (Fig. 1), with the permission of the Salerno Port Authority. The chemical composition of the water of the Port of Salerno is provided in Table 3, together with that of normal seawater (as given in the DuraCrete report [15]) and the simulated seawater analysed by Roque et al. [25]. It can be seen that the Salerno Port seawater is approximately chemically comparable to normal seawater, while the simulated seawater adopted by Roque et al. [25] has a rather large salt content. The latter was employed to study concrete damage induced by the hygroscopic behaviour of salts in the concrete pores [25]. For the Salerno Port water we measured pH = 8.10, which is just slightly below the basicity of normal seawater (pH = 8.2–8.4, cf. [15]). It is worth remarking that the Salerno Port water was found quite rich of hydrocarbons.

# 3. Results and discussion

# 3.1. Compression strength tests

Compression strength tests were performed on 150 mm cubic specimens, in accordance with the European standard UNI EN 12390-1 [28]. The tests were carried out in force control at a constant rate of 7 kN/s, using a servo-controlled Schenck testing machine (4000 kN capacity). Tables 4 and 5 illustrate the results obtained in terms of specific gravity and cube compressive strength ( $f_{c,cube}$ ) for Lab and Sea specimens, respectively. The same tables also provide the mean value, the standard deviation and the coefficient of variation of  $f_{c,cube}$  for UNRC, RPETFRC/a and RPETFRC/

#### Table 4

Results of compressive strength tests on Lab specimens.

Specimen ID	Specific gravity (kg/m <sup>3</sup> )	f <sub>c,cube</sub>			
		Specimen strength (MPa)	Mean value $\bar{f}_{c,cube}$ (MPa)	Standard deviation (MPa)	Coefficient of variation (%)
UNRC – Lab-1	2222	40.6	42.4	1.6	3.9
UNRC – Lab-2	2311	42.6			
UNRC – Lab-3	2207	43.9			
RPETFRC/a – Lab-1	2299	40.6	40.0	0.8	2.0
RPETFRC/a – Lab-2	2258	39.1			
RPETFRC/a – Lab-3	2214	40.3			
RPETFRC/c – Lab-1	2228	37.6	38.9	1.6	4.0
RPETFRC/c - Lab-2	2252	38.5			
RPETFRC/c - Lab-3	2240	40.6			

Table 5

Results of compressive strength tests on Sea specimens.

Specimen ID	Specific gravity (kg/m <sup>3</sup> )	Cube compressive strengt	h f <sub>c,cube</sub>			$\Delta \bar{f}_{c,cube}$ (%)
		Specimen strength (MPa)	Mean value $\bar{f}_{c,cube}$ (MPa)	Standard deviation (MPa)	Coefficient of variation (%)	
UNRC – Sea/6-1 UNRC – Sea/6-2 UNRC – Sea/6-3	2249 2278 2258	46.6 40.6 40.5	42.6	2.8	6.7	+0.5
UNRC – Sea/12-1 UNRC – Sea/12-2 UNRC – Sea/12-3	2393 2387 2371	47.8 39.3 40.4	42.5	3.8	8.9	+0.3
RPETFRC/a – Sea/6-1 RPETFRC/a – Sea/6-2 RPETFRC/a – Sea/6-3	2216 2202 2205	37.6 39.4 39.6	38.8	0.9	2.3	-2.9
RPETFRC/a – Sea/12-1 RPETFRC/a – Sea/12-2 RPETFRC/a – Sea/12-3	2356 2312 2375	41.8 32.7 40.3	38.3	4.0	10.4	-4.3
RPETFRC/c – Sea/6-1 RPETFRC/c – Sea/6-2 RPETFRC/c – Sea/6-3	2231 2208 2199	35.4 39.4 40.0	38.2	2.0	5.3	-1.7
RPETFRC/c – Sea/12-1 RPETFRC/c – Sea/12-2 RPETFRC/c – Sea/12-3	2297 2342 2342	35.7 40.8 36.5	37.7	2.2	5.9	-3.2

c, which were computed on considering sets of three specimens for each material and curing condition. In the sequel, we agree to denote the mean value of a generic property x by  $\bar{x}$ .

The results relative to Lab specimens highlight that the UNRC, RPETFRC/a and RPETFRC/c analysed in this work exhibit  $\bar{f}_{c,cube}$  equal to 42.4 MPa, 40.0 MPa and 38.9 MPa, respectively (cf. Table 4). We observe a reduction of  $\overline{f}_{c,cube}$  equal to -5.7% in RPETFRC/a and -8.3% in RPETFRC/c, as compared to UNRC. Such results markedly differ from the analogous ones presented by Fraternali et al. [1] for pozzolana cement-based concretes with a 53% water/cement ratio, which showed 35.1% and 0.3% increments of  $\overline{f}_{c,cube}$  in RPETFRC/a and RPETFRC/c, respectively, against UNRC. Ochi et al. [2] recorded 8.4%, 13.8% and 6.0% increases of  $\bar{f}_{c.cube}$  for the R-PET reinforcements at 1% fibre volume fraction of concretes with water/cement ratios equal to 65%, 60% and 55%, respectively (these authors did not mention the nature of the employed binder). Kim et al. [3], on the other hand, observed a -7% decrease of  $f_{c,cube}$  for the R-PET reinforcement at 1% fibre volume fraction of ordinary Portland cement-based concretes with a water/cement ratio equal to 0.41 (always compared to UNRC). It is possible to recognize a general trend indicating a decrease of the RPETFRC compressive strength with the water/cement ratio, for a constant fibre volume ratio (1%). This might imply that the beneficial effects of the R-PET reinforcement in terms of compressive strength are more pronounced in the presence of low-strength-class concretes.

The compressive strengths of Sea specimens are shown in Table 5, where  $\Delta \bar{f}_{c,cube}$  denotes the difference between the values of  $\bar{f}_{c,cube}$ corresponding to Sea and Lab specimens, for each examined material (UNRC, RPETFRC/a and RPETFRC/c). We observe slight increases of  $\bar{f}_{c,cube}$  in UNRC and appreciable decreases of the same quantity in RPETFRC/a and RPETFRC/c, due to seawater curing. The decreases of  $\bar{f}_{c,cube}$  in RPETFRC/a and RPETFRC/c grow with the seawater exposure time, and amount to -4.3%, for RPETFRC/a, and -3.2%, for RPETFRC/c, after 12 months. We also observe decreases of  $\bar{f}_{c.cube}$ in seawater-conditioned RPETFRC/a and RPETFRC/c over seawater-conditioned UNRC (-9.9% and -11.3%, respectively). These results confirm the slightly negative effect of R-PET fibres in terms of FRC compressive strength, which was already noticed at 28 days of air-curing (Lab specimens), for the concrete mixes analysed in the present work. It is worth noting that  $f_{c,cube}$  remains approximately constant in UNRC after 6 and 12 months of seawater exposure (cf. Table 5). The same quantity, however, slightly decreases with the seawater curing time in RPETFRC/a and RPETFRC/c. More specifically, RPETFRC/a shows a progressive decrease of the compressive strength after 6 months (-2.9%) and 12 months (-4.3%)of submersion. Less relevant, but still progressive, is the degradation of  $\overline{f}_{c,cube}$  in RPETFRC/c after 6 months (-1.7%) and 12 months (-3.2%) of submersion. Won et al. [26] observed marked decreases in the compressive strength of an FRC showing a 50% water/cement ratio and embossed R-PET fibres at 1% volume fraction, as a



Fig. 2. Four-point bending test set-up and instrumentation.

consequence of the prolonged exposure of the material to markedly alkaline (pH = 12.6) or sulphuric acid  $(3\% H_2SO_4)$  environments.

# 3.2. Four-point bending tests

Four-point bending tests were performed on three prismatic 150 mm  $\times$  150 mm  $\times$  600 mm specimens for each examined

material (UNRC, RPETFRC/a and RPETFRC/c) according to the standards UNI 11039-1 [29] and UNI 11039-2 [30]. Each tested specimen was first notched at the midspan (notch width equal to 2 mm at the mouth; notch depth  $a_0$  equal to 45 mm, cf. UNI EN 11039-2 [30]). The crack tip opening displacement (CTOD) was measured through two displacement transducers placed on the opposite faces of the specimen (denoted as CTOD1 and CTOD2 in Fig. 1), in correspondence with the crack tip. Hereafter, we denote the mean value of the displacements measured by such transducers by CTOD<sub>m</sub> (mean crack tip opening displacement). The midspan deflection *d* was measured through a vertical displacement transducer (denoted as DT in Fig. 3). A 50 kN load cell was used to measure the total load *P* applied to the top surface of the specimen. The test set-up and the employed instrumentation are shown in Figs. 2 and 3.

Tables 6 and 7 illustrate the results of bending tests in terms of the peak load  $P_{\text{max}}$ ; the CTOD<sub>m</sub> corresponding to  $P_{\text{max}}$  (CTOD<sub>m</sub><sup>*p*<sub>max</sub>); the first-crack load  $P_{lf}$ ; and the first-crack strength  $f_{lf}$ , for Lab and Sea specimens, respectively. These tables also provide the mean values of  $P_{\text{max}}$  and  $f_{lf}$  recorded for UNRC, RPETFRC/a and RPETFRC/c. In line with UNI 11039-2 [30], we let CTOD<sub>0</sub> denote</sup>



Fig. 3. Instrumented RPETFRC specimen subjected to four-point bending test. (a) Initial configuration (zero applied load); (b) configuration after crack propagation.

# Table 6

Results of four-point bending tests on Lab specimens.

Specimen ID	$P_{\rm max}$ (N)	$CTOD_m$ at $P_{max}$ (mm)	Mean value of $P_{\text{max}}$ (N)	$P_{lf}$ (N)	$f_{lf}$ (MPa)	Mean value of $f_{lf}$ (MPa)
UNRC – Lab-1	13,366	0.084	13,910	13,289	3.62	3.77
UNRC – Lab-2	15,241	0.079		15,144	4.12	
UNRC – Lab-3	13,124	0.069		13,124	3.57	
RPETFRC/a – Lab-1	13,813	0.076	13,331	13,813	3.76	3.62
RPETFRC/a – Lab-2	12,268	0.078		12,218	3.32	
RPETFRC/a – Lab-3	13,913	0.063		13,919	3.79	
RPETFRC/c – Lab-1	13,591	0.079	15,181	13,520	3.68	4.07
RPETFRC/c – Lab-2	16,740	0.086		16,680	4.54	
RPETFRC/c – Lab-3	15,212	0.084		14,720	4.01	

# Table 7

Results of four-point bending tests on Sea specimens.

Specimen ID	$P_{\rm max}$ (N)	$CTOD_m$ at $P_{max}$ (mm)	Mean value of $P_{\text{max}}$ (N)	$P_{If}$ (N)	$f_{lf}$ (MPa)	Mean value of $f_{lf}$ (MPa)
UNRC – Sea/12-1	13,701	0.084	14,316 (+2.9%)	13,701	3.73	3.90 (+3.4%)
UNRC – Sea/12-2	15,361	0.068		15,361	4.18	
UNRC – Sea/12-3	13,887	0.090		13,887	3.78	
RPETFRC/a – Sea/12-1	14,375	0.059	13,590 (+1.9%)	14,137	3.85	3.61 (-0.4%)
RPETFRC/a – Sea/12-2	12,738	0.089		12,659	3.44	
RPETFRC/a – Sea/12-3	13,656	0.045		12,989	3.53	
RPETFRC/c - Sea/12-1	15,164	0.118	14,999 (-1.2%)	15,147	4.12	4.01 (-1.5%)
RPETFRC/c – Sea/12-2	16,197	0.081		16,197	4.41	
RPETFRC/c - Sea/12-3	13,636	0.081		12,885	3.51	

the mean value of  $\text{CTOD}_m^{P_{\text{max}}}$  for the UNRC (plain concrete), and we define  $P_{lf}$  as the maximum load observed in the  $\text{CTOD}_m$  range [0,CTOD0]. The first-crack strength is computed on the basis of the following formula (UNI 11039-2 [30]):

$$f_{lf} = \frac{P_{lf}l}{b(h-a_0)^2}$$
(1)

where l is the clear span of the specimen, while b and h are the width and the height of the specimen cross section, respectively (Fig. 2).

In the case of Lab specimens, the results presented in Table 6 show a slight decrease of  $P_{max}$  (-4.3%) and  $f_{if}$  (-2.7%), when passing from UNRC to RPETFRC/a. In contrast, RPETFRC/c exhibits remarkable increases of  $P_{max}$  (+9.5%) and  $f_{if}$  (+8.0%) over UNRC (we found CTOD<sub>0</sub> = 0.077 mm for Lab specimens). It is worth noting that PET/a fibres approximately show twice greater tensile strength than PET/c fibres (550 MPa vs. 274 MPa, cf. Table 1). These results suggest that the fibre waviness might be more beneficial than the fibre strength in terms of the flexural strength of the current FRCs, since PET/c fibres have a crimped profile, while PET/a fibres have a

(a) 20000

18000

16000

straight profile. It is worth noting that an opposite trend was observed by Fraternali et al. [1], for what concerns the R-PET reinforcement of a pozzolana cement-based concrete with a 0.53 water/cement ratio. As a matter of fact, for such a concrete, the flexural strength of RPETFRC/a was found to be about 40% greater than that of UNRC, while the flexural strength of RPETFRC/c was found to be about 8% greater than the  $f_{lf}$  of UNRC [1]. Interestingly, the increase in  $f_{if}$  of RPETFRC/c (over UNRC) remains approximately the same for the concrete analysed in the present work and that analysed by Fraternali et al. [1]. Ochi et al. [2] recorded 7.9%, 2.2% and 15.2% increases of the bending strength of fibre-reinforced concretes with 1% R-PET fibre volume fraction and water/cement ratios equal to 65%, 60% and 55%, respectively (over UNRC). Kim et al. [3] observed a 32% increase in the value of  $P_{\text{max}}$  of 200 mm  $\times$  300 mm  $\times$  2000 mm concrete specimens reinforced by R-PET fibres at 1% volume fraction plus steel rebars, as compared to concrete specimens reinforced by rebars only (mix design based on an ordinary Portland cement and a 0.41 water/cement ratio). Interestingly, the representative load-deflection curve of concrete specimens reinforced by rebars only shows a slightly greater turning point  $(P_{cr})$  than the representative load-displacement

-UNRC-Lab-1

-UNRC-Lab-2

UNRC-Lab-3

RPETFRC/a-Lab-1 14000 RPETFRC/a-Lab-2 RPETFRC/a-Lab-3 12000 RPETFRC/c-Lab-1 P (N 10000 RPETFRC/c-Lab-2 RPETFRC/c-Lab-3 8000 6000 4000 2000 0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 d (mm) (b) 20000 UNRC-Sea/12-1 18000 UNRC-Sea/12-2 UNRC-Sea/12-3 16000 RPETFRC/a-Sea/12-1 14000 RPETFRC/a-Sea/12-2 RPETFRC/a-Sea/12-3 12000 RPETFRC/c-Sea/12-1 D N RPETFRC/c-Sea/12-2 10000 RPETFRC/c-Sea/12-3 8000 6000 4000 2000 0 0.0 0.5 1.0 1.5 2.0 25 3.0 3.5 40 4.5 50 d (mm) Fig. 4. Total applied load vs. midspan displacement curves: (a) Lab specimens; (b) Sea specimens.

curve of specimens reinforced by 1% R-PET fibre volume fraction plus steel rebars (cf. Table 4 and Fig. 11 of [3]).

We now examine the results of bending tests on Sea specimens, quoting in brackets the variations of  $\bar{P}_{max}$  and  $\bar{f}_{lf}$  when passing from Sea to Lab specimens. The results in Table 7 show appreciable increases in the UNRC flexural strength, and slight variations in the RPETFRC/a and RPETFRC/c flexural strengths, due to seawater conditioning (we measured CTOD<sub>0</sub> = 0.081 mm for Sea specimens, after 12 months of seawater curing). Compared to seawater-conditioned UNRC, we observe decreases of  $P_{max}$  (-5.1%) and  $f_{lf}$  (-7.4%) in RPETFRC/a, and increases of  $P_{max}$  (+4.8%) and  $f_{lf}$  (+3.0%) in RPETFRC/c.

Fig. 4(a) and (b) illustrates the P-d curves obtained for Lab and Sea/12 specimens, respectively. Fig. 5(a) and (b), on the other hand, illustrates the  $P-\text{CTOD}_m$  curves obtained for RPETFRC/a Lab and Sea specimens, respectively, while Fig. 6(a) and (b) shows the analogous curves obtained for RPETFRC/c. The above plots highlight a markedly brittle response of UNRC Lab and Sea specimens, which exhibit almost zero energy absorption capacity once the first-crack load is reached (cf. Fig. 4a and b), and an appreciable reduction of

the energy absorption capacity of seawater-conditioned RPETFRC/a and RPETFRC/c specimens, as compared to the analogous Lab specimens (Figs. 4–6).

In order to analyse the post-cracking behaviour of RPEFRC/a and RPETFRC/c, we now introduce the following energies,  $U_1$  and  $U_2$ , which are associated with the  $\text{CTOD}_m$  ranges [CTOD<sub>0</sub>,  $\text{CTOD}_0 + 0.6 \text{ mm}$ ] and [CTOD<sub>0</sub> + 0.6 mm,  $\text{CTOD}_0 + 3 \text{ mm}$ ] of the  $P - \text{CTOD}_m$  response, respectively (UNI 11039-2 [30]):

$$U_{1} = \int_{\text{CTOD}_{0}+0.6 \text{ mm}}^{\text{CTOD}_{0}+0.6 \text{ mm}} P(\text{CTOD}_{m}) d \text{CTOD}_{m},$$
  

$$U_{2} = \int_{\text{CTOD}_{0}+0.6 \text{ mm}}^{\text{CTOD}_{0}+3 \text{ mm}} P(\text{CTOD}_{m}) d \text{CTOD}_{m}$$
(2)

In line with UNI 11039-2 [30], we characterize the energy absorption capacity of the examined FRCs through the following "ductility indices"  $D_0$  and  $D_1$ :

$$D_0 = \frac{f_{eq(0-0.6)}}{f_{lf}}; \quad D_1 = \frac{f_{eq(0.6-3)}}{f_{lf(0-0.6)}}; \tag{3}$$



Fig. 5. Total applied load vs. CTDO<sub>m</sub> curves for RPETFRC/a: (a) Lab specimens; (b) Sea specimens.

where  $f_{eq(0-0.6)}$  and  $f_{eq(0.6-3)}$  are the following equivalent stresses associated with the  $CTOD_m$  ranges  $[CTOD_0, CTOD_0 + 0.6 \text{ mm}]$  and [CTOD<sub>0</sub> + 0.6 mm, CTOD<sub>0</sub> + 3 mm], respectively:

$$f_{eq(0-0.6)} = \frac{l}{b(h-a_0)^2} \frac{U_1}{0.6}; \quad f_{eq(0.6-3)} = \frac{l}{b(h-a_0)^2} \frac{U_2}{2.4}.$$
 (4)

The above indices can be understood as dimensionless indicators of the nature of the post-crack response (or "ductility class") of the FRC, since "hardening" post-cracking behaviour corresponds to values greater than one of such quantities, while "softening" behaviour corresponds to values smaller than one. In particular,  $D_0$  characterizes the "ductility" of the material (defined as above) in the CTOD<sub>m</sub> range that immediately follows crack onset ("firstcrack ductility"), while  $D_1$  characterizes the material "ductility" in a heavily cracked regime ("ultimate ductility"). Table 8 summarizes the results obtained in the present work in terms of energy absorption capacities and ductility indices of the examined **RPETFRCs.** 

With regard to air-cured (Lab) specimens, it is seen that RPETFRC/a exhibits average values of  $D_0$  and  $D_1$  equal to 0.46 and

0.48, respectively, while RPETFRC/c exhibits average values of  $D_0$ and  $D_1$  equal to 0.32 and 0.43, respectively. The RPETFRC/a analysed in [1], however, showed  $D_0 = 0.82$  and  $D_1 = 0.68$ , while the RPETFRC/c analysed in the same work featured  $D_0 = 0.95$  and  $D_1 = 0.58$  (R-PET fibre reinforcement at 1% fibre volume content of a pozzolana cement-based concrete with a 0.53 water/cement ratio). The RPETFRCs analysed in the present work exhibit much lower ductility indices (and, consequently, much lower fracture toughness) than the RPETFRCs studied in [1]. We also notice that RPEFTRC/a exhibits higher ultimate ductility than RPETFRC/c, both in the present case and in [1] (cf. Figs. 4 and 5 of the present work, and Fig. 5 of [1]). This can be explained by a fibre-debonding-type failure mode of RPETFRC/a (due to the high strength and straight aspect of PET/a fibres), and a fibre-rupture-type failure mode of RPETFRC/c (due to the relatively low tensile strength of PET/c fibres, and the crimped aspect of such fibres). Ochi et al. [2] measured relative energy capacities (average energies absorbed by the analysed RPETFRCs divided by the average energy absorbed by the unreinforced concrete) equal to 5.14, 5.70 and 5.95 for RPETFRCs with 1% R-PET fibre volume fraction and water/cement

P (N 10000 8000 6000 4000 2000 0 0.5 1.0 0.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 CTODm (mm) Fig. 6. Total applied load vs CTDO<sub>m</sub> curves for RPETFRC/c: (a) undisturbed specimens; (b) conditioned specimens.



Table 8
Energy absorption capacities and ductility indices of the examined RPETFRC specimens.

Specimen ID	[CTOD <sub>0</sub> , CTOD <sub>0</sub> + 0.6 mm]			[CTOD <sub>0</sub> + 0.6	[CTOD <sub>0</sub> + 0.6 mm, CTOD <sub>0</sub> + 3 mm]			Total post-crack energy		
	$U_1$ (Nmm)	$f_{eq(0-0.6)}$ (MPa)	$D_0$	$\overline{D_0}$	$U_2$ (Nmm)	$f_{eq(0.6-3)}$ (MPa)	$D_1$	$\overline{D_1}$	$U_1 + U_2$ (Nmm)	$\overline{U_1 + U_2}$ (Nmm)
RPETFRC/a – Lab-1 RPETFRC/a – Lab-2	3217 3338	1.46 1.51	0.39 0.46	0.46	11608 12,712	1.32 1.44	0.35 0.43	0.48	14,825 13,855	19,187
RPETFRC/a – Lab-3	4530	2.05	0.54		22,155	2.51	0.66		21,579	
RPETFRC/c – Lab-1 RPETFRC/c – Lab-2 RPETFRC/c – Lab-3	3443 2273 2805	1.56 1.03 1.27	0.42 0.23 0.32	0.32	15,487 13,804 16,008	1.76 1.57 1.81	0.48 0.34 0.45	0.43	15,973 13,607 15,326	17,940
RPETFRC/a – Sea/12-1 RPETFRC/a – Sea/12-2 RPETFRC/a – Sea/12-3	3934 3103 2730	1.78 1.41 1.24	0.46 0.41 0.35	0.41	10,230 5001 7007	1.16 0.57 0.79	0.30 0.16 0.22	0.23	14,164 8104 9737	10,668
RPETFRC/c – Sea/12-1 RPETFRC/c – Sea/12-2 RPETFRC/c – Sea/12-3	3307 3456 4206	1.50 1.57 1.91	0.36 0.36 0.54	0.42	4636 6903 9480	0.53 0.78 1.07	0.13 0.18 0.31	0.20	7943 10,359 13,686	10,663

Table 9

ratios equal to 65%, 60% and 55%, respectively. Kim et al. [3] observed a relative energy capacity equal to 4.34 for concrete specimens reinforced by 1% R-PET fibre volume fraction and steel rebars (ordinary Portland cement-based concrete with a 0.41 water/cement ratio).

We now move to examining the post-cracking behaviour of seawater-cured (Sea) specimens. The results in Table 8 indicate that the seawater conditioning lowers all the ductility indices of RPETFRC/a. This reduction is light in terms of the first-crack ductility, since  $\bar{D}_0$  shrinks from 0.46 (Lab specimens) to 0.41 (Sea specimens) for such a material. The loss in the ultimate ductility of RPETFRC/a is, however, rather pronounced, since  $\bar{D}_1$  reduces from 0.48 (Lab) to 0.23 (Sea). With regard to RPEFTRC/c, we observe an increase of  $\bar{D}_0$  passing from Lab (0.32) to Sea (0.42) specimens, and a marked decrease of  $\bar{D}_1$  due to seawater conditioning, since such a "ductility" index is equal to 0.43 for Lab specimens, and 0.20 for Sea specimens.

It is worth comparing the flexural responses of RPETFRC/a and RPETFRC/c Sea/12 specimens with the analogous ones of FRCs reinforced with PP (0.5% volume fraction), PVA (0.75% volume fraction) and Steel fibres (1.0% volume fraction), and subjected to 21 months' conditioning in simulated seawater ([25], cf. Table 3). We carry out such a comparison by introducing the following toughness index ([25], ASTM C1609 [31]):

$$T_{150} = \int_0^{L/150} P(d) \, \mathrm{d}d,\tag{5}$$

where *d* is the midspan deflection, and *L* is the clear span of the specimen. We also compare the mean values of the cylindrical compressive strengths  $f_c$  of the above materials, making use of the following conversion formula  $f_{c,cube} \approx 0.83 f_c$  [32].

Table 9 shows mean values of  $f_c$  for the FRCs analysed in this work and [25], and the differences  $\Delta f_{lf}$  and  $\Delta \overline{T}_{150}$  between the values of  $\bar{f}_{lf}$  and  $\bar{T}_{150}$  of seawater- and air-cured specimens, respectively. We observe small values of  $\Delta \bar{f}_{lf}$  in all the examined FRCs, which indicates that the first-crack strength is not strongly influenced by seawater conditioning (cf. also Table 7). The case of the flexural toughness is very different, since we observe decreases of such a quantity in all the examined FRCs, as a result of prolonged seawater conditioning. In particular, PVA-FRC features the best resistance to seawater degradation among the examined materials, showing only a 4% decrease in  $\overline{T}_{150}$  after 21 months of seawater conditioning. Marked decreases of such a quantity are, conversely, featured by PP-FRC (-27%), Steel-FRC (-25%), RPETFRC/a (-42%) and RPETFRC/c (-38%), after prolonged seawater curing (21 months in the case of PP-FRC and Steel-FRC; 12 months in the case of RPETFRC/a and RPETFRC/c).

Seawater	degradation	of	different	FRCs.

	Mean value of cylindrical compressive strength $\bar{f}_c$ (MPa)	First-crack strength variation $\Delta \bar{f}_{lf}$ (%)	Toughness variation $\Delta \bar{T}_{150}$ (%)
PP-FRC (0.5%) [25]	55.8	-5.8	-26.7
PVA-FRC (0.75%) [25]	58.0	5.4	-4.2
Steel-FRC (1.0%) [25]	63.8	-2.8	-24.5
RPETFRC/a – Sea/12 (1%)	33.3	1.9	-41.7
RPETFRC/c – Sea/12 (1%)	32.3	-1.2	-37.9

# 4. Concluding remarks

We have presented an experimental study on the mechanical properties of Portland limestone cement-based concretes showing 0.38 water/cement ratio and reinforcing R-PET fibres at 1% fibre volume content. The results of compression tests and four-point bending tests on plain concrete (UNRC) and two different RPETFRCs have been presented and reviewed with reference to both 28 days' air-cured (Lab) and 6/12 months' seawater-cured (sea) samples.

Concerning air-cured specimens, the results presented in this work and previous literature [1-3] demonstrate that the R-PET fibre reinforcement of concrete is highly beneficial in terms of the energy absorption capacity of the material. In particular, the concrete analysed in the present work showed first-crack and ultimate 'ductility' indices  $\overline{D_0}$  and  $\overline{D_1}$  (UNI 11039-2 [30]) respectively equal to 0.46 and 0.48, when reinforced with straight and high-strength R-PET fibres (PET/a). The same concrete reinforced with crimped and relatively low-strength R-PET fibres (PET/c) showed  $\overline{D_0}$  and  $\overline{D_1}$  respectively equal to 0.32 and 0.43. These marked increments in the material toughness compared with plain concrete (UNRC), which exhibits  $\overline{D_0}$  and  $\overline{D_1}$  approximately equal to zero, were accompanied by a slight decrease in the first-crack strength  $\overline{f}_{if}$ (-3.9%) in RPETFRC/a, and, in contrast, by a remarkable increase of  $\overline{f}_{lf}$  (+8.0%) in RPETFRC/c, always compared with UNRC. Both RPETFRC/a and RPETFRC/c showed decreases of the compression strength  $\bar{f}_{c.cube}$  compared with UNRC which were respectively equal to -5.7% and -8.3%. By comparing the above results with similar ones presented elsewhere [1,3], we recognize a general trend indicating a reduction of the beneficial effects of R-PET fibres with the water/cement ratio of concrete. A comparative analysis between the present study and that of Fraternali et al. [1] also indicates that PET/a fibres appear to be highly beneficial in terms of compressive and tensile strength properties in the case of pozzolana cementbased concretes with a high water/cement ratio (0.53). In contrast, PET/c fibres appear to be well suited to improving the compressive strength and first-crack strength of Portland limestone cement-based concretes with a low water/cement ratio (0.38).

With regard to seawater curing of RPETFRC, we observed that prolonged submersion of the analysed materials in a reserved area of the Port of Salerno (up to 12 months) led to minor modifications of the compressive strength and first-crack strength of the material, as compared with the air-cured counterpart. On the other hand, we have observed marked reductions of the energy absorption capacity of the analysed RPETFRCs after 12 months of seawater conditioning, again compared with air-cured materials. Such significant deterioration in toughness essentially takes place in the heavily cracked regime (reduction of  $D_1$  equal to -52.1% in RPETFRC/a and -5.5% in RPETFRC/c), whereas  $D_0$  either moderately decreases (RPETFRC/a: -10.9%) or increases (RPETFRC/c: +31.3%) because of seawater conditioning.

Overall, the results here obtained suggest that the addition of R-PET fibres to cementitious materials needs to be accurately tailored in the RPETFRC design mix, with regard to the choice of the fibre properties, the desired strength and the ductility properties of the final material (cf. also [9] with regard to the R-PET reinforcement of cement mortars).

An extension of the present work focused on the durability of a variety of RPETFRCs in the presence of different mix designs and curing conditions is required. We also recommend the mechanical modelling of crack initiation and propagation in RPETFRC structural elements, to be conducted by means of free-discontinuity and/or eigenfracture variational models [33,35,34,36,37]. Such research can be expected to provide useful theoretical information, validated by experimental studies, about the durability of R-PET concrete reinforcement under severe environmental conditions and the load-carrying capacity of RPETFRC structural members under service and ultimate loads.

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