MULTIAXIAL PRESTRESS OF REINFORCED CONCRETE I-BEAMS

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SUMMARY- We analyze the effects of multiaxial prestress on the limit behavior of I-shaped reinforced concrete elements subjected to combined axial load and bending. We provide a collection of design charts and moment – curvature diagrams for reinforced concrete I-beams with laterally and longitudinally prestressed flanges. The given results highlight the special ability of the active prestress technique in enhancing the strength and ductility properties of seismic resistant columns and shear walls.

Keywords: lateral prestress, I-beams, prestressed reinforced concrete, strength domains, moment-curvature diagrams, structural ductility.

1. Introduction

The lateral confinement of reinforced concrete (RC) elements through pre-heated steel rings, ordinary stirrup reinforcements, and/or Fiber-Reinforced Polymer (FRP) wraps are well-known strengthening techniques in the field of construction industry. Such reinforcement methods are able to transform the uniaxial state of stress of an unconfined element, which is subject to axial loading and bending, into a multiaxial state of stress, due to the restraining pressure applied by the transverse reinforcement. The latter may appreciably increase the material strength, by moving Mohr’s stress circles towards the compression region of the intrinsic strength curve (Fig. 1).

![Mohr circles](image)

Fig. 1 Strengthening effect of lateral prestress in brittle materials: shifting of Mohr circles towards the compression region (left) of the intrinsic strength curve.

The “passive” confinement of RC elements is mainly effective in presence of large axial strains (i.e., in correspondence with axial stresses approaching the concrete uniaxial strength), and less effective under small axial strains, due to low Poisson’s ratios of common concretes.
More challenging is the “active” confinement of I-shaped RC members, which is obtained by laterally prestressing the terminal flanges (Fig. 2). Such a prestressing technique is well suited for seismic resistant RC columns or shear walls, where the presence of bi-axial/tri-axial compression states may significantly enhance strength and ductility properties of the element (Fig. 3).

Fig. 2 Active lateral prestress of the flanges of I-beams.

Fig. 3 Regions to be laterally prestressed in a RC shear wall: a) horizontal cross-section; b) vertical cross-section.

[Gardener et al., 1992] present an experimental and theoretical study on the load capacity of laterally compressed RC columns subjected to eccentric axial forces, formulating an empirical formula to predict the ultimate load of such elements. [Zaki, 2013] studies the optimal design of external FRP confinements of RC columns subject to axial and bending loads, determining optimized values of the FRP wrapping thickness and lengths. [Saadatmanesh et al., 1997] present an experimental study on the repair though FRP wraps of earthquake damaged RC columns. The FRP reinforced columns examined in such a study show enhanced hysteretic response, flexural strength and displacement ductility, as compared to the original elements. [Janke et al., 2009] investigate on the compression behavior of cylindrical concrete elements reinforced with prestressed steel and carbon FRP wraps. The outcomes of this study show that the residual capacity of the elements reinforced with prestressed confinements is significantly
higher than that of elements reinforced with unstressed wraps, even in presence of moderate confinement prestress. The strengthening and rehabilitation of RC members through external wrapping with prestressed FRP elements is investigated by [Mortazavi et al., 2003] and [Mukherjee and Rai, 2009]. Such studies show that the prestressing of confinement wraps and laminates significantly enhances the utilization of FRP materials and the concrete response [Mortazavi et al., 2003], as well as the design and ultimate loads (by more than 100%) of FRC rehabilitated beams [Mukherjee and Rai, 2009].

The present work examines the ultimate limit state design of RC I-beams featuring 3D prestressed flanges, and subject to combined axial loads and bending moments. We deal with a parametric study on the strength domains and moment-curvature diagrams of such elements, on examining different aspect ratios, prestress levels, and steel reinforcement ratios. Our final goal is to show the effects of such design variables on the load-carrying capacity of I-shaped RC elements subject to multiaxial precompression.

The remainder of the paper is organized as follows. We describe in Sect. 2 the adopted constitutive laws for concrete, under triaxial stress states, and ordinary and high-strength steel rebars. Next, we present a variety of design charts (Sect. 3), and moment-curvature diagrams (Sect. 4) of laterally prestressed I-beams, examining different cross-section geometries, lateral prestress ratios, longitudinal prestrain levels, and steel reinforcement ratios. The given results emphasize the special ability of the lateral prestress technique in enhancing both strength and ductility properties of the examined RC elements, even under moderately large lateral prestress of the cross-section flanges. The main conclusions and future research developments are proposed in Sect. 5.

2. Material and cross-section modeling

Let us examine the mechanical response of a I-shaped concrete member with respect to an orthogonal reference frame \( x_1, x_2, x_3 \), which has the \( x_1 \) and \( x_2 \) axes aligned with the principal directions of the cross-section (\( x_1 \) parallel to the flanges, and \( x_2 \) parallel to the web), and the \( x_3 \) axis aligned with the member axis. Such an element is subject to constant lateral prestresses \( \sigma_1 \) and \( \sigma_2 \), and is equipped with ordinary steel rebars along the web, and high-strength steel rebars along the flanges. The latter might eventually be (longitudinally) prestressed, in order to apply a fully 3D prestress state to the member under consideration. The constitutive response of the different elements composing such a member (concrete and reinforcing steel bars) are first modeled. The overall cross-section response is then described.

2.1 Failure surface of concrete

Assuming isotropic behavior, the failure (or yield) surface of concrete under triaxial stress states can be drawn in the 3D Haigh–Westergaard space of the principal stresses, \( \sigma_1, \sigma_2, \sigma_3 \) and characterized through [Ferrara et al., 1977; Ottosen, 1979; Bigoni and Piccolroaz, 2004; Higgins et al., 2013; Gupta, 2013; Liolios and Exadaktylos, 2013]

- “deviatoric” sections with planes orthogonal to the hydrostatic axis \( \sigma_1 = \sigma_2 = \sigma_3 \) (deviatoric planes, Fig. 4);
- “meridian” sections with Rendulic planes [Ferrara et al., 1977] (shear stress-mean stress planes, see Fig. 5).
Such a surface is usually described in terms of the first invariant of the stress tensor, and the second and third invariants of the deviatoric stress tensor, i.e. the quantities

\[ I_\sigma = \sigma_1 + \sigma_2 + \sigma_3; \quad J_{2\sigma} = \frac{1}{2}(\bar{\sigma}_1^2 + \bar{\sigma}_2^2 + \bar{\sigma}_3^2); \quad J_{3\sigma} = \bar{\sigma}_1 \bar{\sigma}_2 \bar{\sigma}_3, \quad (1) \]

where

\[ \bar{\sigma}_1 = \sigma_1 - \sigma_{\text{oct}} \]
\[ \bar{\sigma}_2 = \sigma_2 - \sigma_{\text{oct}} \]
\[ \bar{\sigma}_3 = \sigma_3 - \sigma_{\text{oct}} \]

\[ \sigma_{\text{oct}} = \frac{I_{3\sigma}}{3}. \quad (2) \]

Fig. 4: a) current deviatoric section of the failure surface; b) deviatoric section far away from the origin of the principal stress space; c) deviatoric section close to the origin of the principal stress space.

Fig. 5: Longitudinal section of the failure surface with a Rendulic plane.

A popular yield criterion for concrete has been formulated by [Ottosen, 1977], assuming (Figs. 4,5):

- the failure surface is smooth and convex everywhere, except at the vertex;
- the meridians are parabolic;
- the deviatoric sections switch from quasi-triangular to circular, as the hydrostatic stress increases.

The yield locus formulated by Ottosen depends on four dimensionless parameters \((A, B, K_1, K_2)\) and is defined through the scalar equation
\[
\frac{AJ_{3\sigma}}{|f_c|} + \lambda \frac{J_{2\sigma}}{|f_c|} + \frac{BI_{1\sigma}}{|f_c|} - 1 = 0.
\]

where

\[
\lambda = \begin{cases} 
K_1 \cos \left( \frac{1}{3} \arccos \left( K_2 \cos 3\theta \right) \right) & \text{if } \cos 3\theta \geq 0 \\
K_1 \cos \left( \frac{\pi}{3} - \frac{1}{3} \arccos \left( -K_2 \cos 3\theta \right) \right) & \text{if } \cos 3\theta < 0
\end{cases}
\]

\[
\cos 3\theta = \left( \frac{3}{2} \sqrt{\frac{J_{3\sigma}}{J_{3/2}^2}} \right)
\]

By definition, the parameters \((A, B, K_1)\) are positive, and it results \(0 \leq K_2 \leq 1\). Such quantities can be experimentally determined through the following laboratory tests:

- uniaxial compression, giving the uniaxial compression strength \(f_c\);
- uniaxial tension, providing the uniaxial tensile strength \(f_t\), and the strength ratio \(K = f_t/f_c\);
- uniform biaxial compression, giving the uniform biaxial compressive strength \(f_{2c}\);
- failure test at a given stress state along the compressive meridian, providing the quantities: \(\sigma_{oct}/f_c\) and \(\tau_{oct}/f_c\), where \(\sigma_{oct}\) and \(\tau_{oct}\) are the octahedral normal and shear stresses, respectively (\(\sigma_{oct} = I_{3\sigma}/3, \tau_{oct} = \sqrt{2/3J_{2\sigma}}\)).

Table 1 and Table 2 show experimental values of the above quantities, and the associated Ottosen’s parameters, obtained by [Schickert and Winkler, 1977] (used for the simulations presented in the Sects. 3 and 4), and [Linse and Aschl, 1976].

**Table 1 Available strength parameters of concrete.**

<table>
<thead>
<tr>
<th></th>
<th>[Schickert and Winkler, 1977]</th>
<th>[Linse and Aschl, 1976]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) uniaxial compressive strength (</td>
<td>f_c</td>
<td>)</td>
</tr>
<tr>
<td>2) (K = f_t/f_c)</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>3) biaxial compressive strength (</td>
<td>f_{2c}</td>
<td>)</td>
</tr>
<tr>
<td>4) (\sigma_{oct}, \tau_{oct})</td>
<td>-88.3 MPa, 57.9 MPa</td>
<td>-50.0 MPa, 43.6 MPa</td>
</tr>
</tbody>
</table>

**Table 2 Ottosen parameters.**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>K_1</th>
<th>K_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Schickert and Winkler, 1977]</td>
<td>3.2244</td>
<td>3.4555</td>
<td>11.1538</td>
<td>0.9962</td>
</tr>
<tr>
<td>[Linse and Aschl, 1976]</td>
<td>2.1272</td>
<td>3.2965</td>
<td>11.5006</td>
<td>0.9900</td>
</tr>
</tbody>
</table>

2.2 Stress-strain response of concrete
Let us consider now the stress-strain response of an infinitesimal element of a laterally prestressed concrete member, assuming strain and stress components positive if the material is compressed. We assume that the lateral prestresses \( \sigma_1 \) and \( \sigma_2 \) remain constant during loading, and let the axial strain \( \varepsilon_3 \) increase up to axial failure.

For what concerns the compressive branch of the axial stress-strain law \( (\sigma_3 \text{ vs } \varepsilon_3) \), we discard Poisson’s effect \( (\nu \approx 0) \), and make use of the constitutive law proposed by [Ottosen, 1979], which naturally complements the failure surface discussed in the previous section. Other models of the same (secant) type have been proposed by [Cedolin and Mulas, 1971], and [Cedolin et al., 1977]. Ottosen’s constitutive model depends on the following quantities:

- initial value of the Young modulus \( E_0 \);
- uniaxial compressive and tensile strengths \( f_c \) and \( f_t \);
- “softening” parameter \( D \), which characterizes the slope of the softening branch of the stress-strain response;
- “nonlinearity” factor \( \beta \) defined as follows
  \[
  \beta = \frac{\sigma_3}{\sigma_{3f}},
  \]
  where:
  - \( \sigma_3 \) is the current maximum compressive stress;
  - \( \sigma_{3f} \) is the corresponding failure stress.

The conditions: \( 0 \leq \beta \leq 1, \ \beta = 1, \ \text{and} \ \beta > 1 \), respectively define stress states lying inside, on the boundary, and outside the failure surface. Ottosen’s law for the secant Young modulus of concrete is the following

\[
E_s = \frac{1}{2} E_0 - \beta \left( \frac{1}{2} E_0 - E_f \right) \pm \sqrt{\left( \frac{1}{2} E_0 - \beta \left( \frac{1}{2} E_0 - E_f \right) \right)^2 + E_f^2 \beta \left[ D(1-\beta)-1 \right]}
\]

Equation (7), \( E_f \) denotes the failure value of \( E_s \), which is defined through

\[
E_f = \frac{E_c}{1 + (4(\alpha - 1)k)}.
\]

Here, \( E_c \) denotes the failure value of \( E_s \) under uniaxial compression, and it results

\[
k = \left( \sqrt{\frac{J_{2\sigma}}{f_c}} \right)_f - \frac{1}{3},
\]

\[
\alpha = \frac{E_0}{E_c}.
\]

where \( \left( \sqrt{J_{2\sigma}} \right)_f \) denotes the failure value of the second invariant of the deviatoric stress tensor. For \( k < 0 \), the result \( E_f = E_c \) applies.
For what concerns the tensile branch of the \( \sigma_3 \) vs \( \varepsilon_3 \) response, we assume the tension-stiffening behavior illustrated in Fig. 6 [Belarbi and Hsu, 1994].

The overall longitudinal response of the laterally prestressed element is composed of the following branches

\[
\sigma_3 = \begin{cases} 
\frac{E_0 + \bar{E}_f}{\sigma_{3f}} \frac{\varepsilon_3}{\varepsilon_{3f}} (D - 1) & \varepsilon_{3u} \leq \varepsilon_3 \leq 0 \\
1 + 2 \frac{\varepsilon_3}{\sigma_{3f}} \left( \frac{1}{2} E_0 - \bar{E}_f \right) + \frac{\bar{E}_f^2}{\sigma_{3f}^2} & 0 \leq \varepsilon_3 \leq \varepsilon_{uf} \\
E_0 \varepsilon_3 - (2E_0 \varepsilon_{uf} - 3f_t) \left( \frac{\varepsilon_3}{\varepsilon_{uf}} \right)^2 + (E_0 \varepsilon_{uf} - 2f_t) \left( \frac{\varepsilon_3}{\varepsilon_{uf}} \right)^3 & 0 \leq \varepsilon_3 \leq \varepsilon_{uf} \\
\sigma_3 = f_t & \varepsilon_{uf} \leq \varepsilon_3 \leq \varepsilon_{tu} \\
\sigma_3 = \frac{\bar{\varepsilon}}{\varepsilon_3} & \varepsilon_3 > \varepsilon_{tu}
\end{cases}
\]

(10)

The numerical results presented in the following sections are based on the above constitutive laws, and the material parameters provided in Table 3.

### Table 3 Adopted constitutive parameters of concrete.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_0 )</td>
<td>2.84x10^4 MPa</td>
</tr>
<tr>
<td>( D )</td>
<td>0.2</td>
</tr>
<tr>
<td>( \beta_0 = \beta_1 = \beta_2 = 1 )</td>
<td></td>
</tr>
<tr>
<td>( \varepsilon_{cf,1} )</td>
<td>0.00197</td>
</tr>
<tr>
<td>( \varepsilon_{3f,1} )</td>
<td>0.3%</td>
</tr>
<tr>
<td>( \varepsilon_{uf} = \varepsilon_{tu} = 0.014% )</td>
<td></td>
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</tbody>
</table>

Fig. 6 Longitudinal stress-strain response of a laterally prestressed concrete element [Ottosen, 1979].
2.3 Stress-strain response of reinforcing steel bars

The adopted constitutive laws for the high-strength (“p”) and ordinary (“f”) steel bars are shown in Fig. 6a and Fig. 6b respectively. Let \( \varepsilon_p \) and \( \sigma_p \) respectively denote the longitudinal strain and longitudinal stress acting in the generic high-strength bar, and let \( \varepsilon_f \) and \( \sigma_f \) respectively denote the longitudinal strain and longitudinal stress acting in the current ordinary steel bar. For the high-strength rebars we make use of the Ramberg-Osgood constitutive model [Ramberg and Osgood, 1943], which is defined through (Fig. 7a)

\[
\sigma_p = E_p \varepsilon_p \quad \text{for} \quad \varepsilon_p < \varepsilon_{pe} \tag{14}
\]

\[
\varepsilon_p = \frac{\sigma_p}{E_p} + e_{pp} \left[ \frac{\sigma_p - \sigma_{pe}}{\sigma_{pp} - \sigma_{pe}} \right]^n \quad \text{for} \quad \varepsilon_{pe} < \varepsilon_p < \varepsilon_{pf} \tag{15}
\]

where:

- \( E_p \) is the Young modulus of “p” bars;
- \( \sigma_{pp} \) is the yield stress of “p” bars;
- \( e_{pp} \) is the strain corresponding to \( \sigma_{pp} \);
- \( (\sigma_{pe}, \varepsilon_{pe}) \) are the coordinates of the final point of linear elastic branch;
- \( (\sigma_{pf}, \varepsilon_{pf}) \) are the coordinates of the failure point;
- \( n \) is a prescribed exponent.

For what concerns the ordinary steel bars, we instead make use of the stress-strain response defined by the following branches (Fig. 7b) [Park and Paulay, 1975]

\[
\sigma_f = E_f \varepsilon_f \quad \text{for} \quad \varepsilon_f < \varepsilon_{f1} \tag{16}
\]

\[
\sigma_f = \sigma_{f1} \frac{\varepsilon_f}{\varepsilon_{f1}} \quad \text{for} \quad \varepsilon_{f1} < \varepsilon_f < \varepsilon_{f11} \tag{17}
\]

\[
\sigma_f = \left[ \sigma_{f1} + E_{fh} \left( \frac{\varepsilon_f - \varepsilon_{f11}}{\varepsilon_{f11}} \right) \right] \frac{\varepsilon_f}{\varepsilon_{f1}} \quad \text{for} \quad \varepsilon_{f11} < \varepsilon_f < \varepsilon_{f12} \tag{18}
\]

\[
\sigma_f = \sigma_{f2} \frac{\varepsilon_f}{\varepsilon_{f2}} \quad \text{for} \quad \varepsilon_{f12} < \varepsilon_f < \varepsilon_{fu} \tag{19}
\]

where:

- \( E_f \) is the Young modulus of “f” bars;
- \( E_{fh} \) is the slope of the hardening branch;
- \( (\sigma_{f1}, \varepsilon_{f1}) \) are the coordinates of the final point of the linear elastic branch;
- \( \varepsilon_{f11} \) and \( \varepsilon_{f12} \) are the strains associated with the initial and final points of the hardening branch;
- \( (\sigma_{f2}, \varepsilon_{fu}) \) are the coordinates of the failure point.
Fig. 7 Adopted constitutive laws for reinforcing steel bars: a) high-strength rebars; b) ordinary rebars.

The simulations presented in Sects. 3 and 4 make use of the above constitutive models, and the numerical data provided in Table 4 and Table 5.

<table>
<thead>
<tr>
<th>Table 4 Material parameters employed for high-strength rebars.</th>
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<tbody>
<tr>
<td>$\sigma_{pf} / f_c = 30$</td>
</tr>
<tr>
<td>$E_p / \sigma_{pf} = 233$</td>
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</table>

<table>
<thead>
<tr>
<th>Table 5 Material parameters employed for ordinary rebars.</th>
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<tbody>
<tr>
<td>$\sigma_{fy} / f_c = 15$</td>
</tr>
<tr>
<td>$\varepsilon_{fy} = 1.0%$</td>
</tr>
</tbody>
</table>

2.4 Cross-section modeling

Let us now consider the overall response of the current cross-section (Fig. 8c), under the action of an axial load $N$, and a bending moment $M$ about the $x_1$ axis. We refer our analysis to the following dimensionless quantities

$$
\nu = \frac{N}{A_c |f_c|}, \quad \mu = \frac{M}{A_c I_c |f_c|},
$$

where $A_c$ denotes the concrete cross-section area.

The present modeling of the cross-section response is based on the following assumptions:

- all the materials are isotropic and homogeneous;
- the cross-sections remains plane and unstretched during deformation;
- there is perfect adhesion at the steel-concrete interface.
which lead us to the ultimate strain profiles shown in Fig. 8b, describing all the possible failure modes of the cross-section: (1: tension failure; 2: balanced failure; 3&4: compression failure).

Fig. 8 Noticeable cross-section strain profiles: a) strain profiles after the prestress of “p” bars; b) ultimate strain profiles; c) cross-section layout.

3. Design charts of laterally prestressed members

We provide in the present section interaction diagrams plotting the dimensionless ultimate axial load \( \nu = N / (A_c f_c) \) against the dimensionless ultimate bending moment \( \mu = M / (A_c l_w f_c) \). Such diagrams have been numerically computed on the basis of the constitutive assumptions introduced in the previous section (Fig. 9). Referring to the cross-section scheme shown in Fig. 9a, we agree to neglect the tensile branch of the concrete stress-strain response (Fig. 6), and to make use of the following aspect ratio

\[
t = t_1 = t_2 = \frac{l_w}{10},
\]

where \( l_w \) denotes the web height. We also consider the following values of the flange widths (see Figs. 9 b-e)

\[
A = A_1 = A_2 = \left[ \frac{l_w}{10}, \frac{l_w}{3}, \frac{l_w}{2}, l_w \right],
\]

and four different values of the lateral prestress

\[
\sigma_c = \sigma_{c1} = \sigma_{c2} = [0, \ 0.10 f_c, \ 0.20 f_c, \ 0.33 f_c].
\]

Finally, we analyze the following reinforcement ratios

\[
\frac{A_p}{A_c} = \frac{A_{p1}}{A_c} = \frac{A_{p2}}{A_c} = 0.00375, \quad A_f = 0.0025.
\]

and three different longitudinal prestrain values of the flange rebar.
The design charts shown in Figs. 9 b-e highlight the influences of the aspect ratio $A/l_w$ and the lateral prestress level $\sigma_c/f_c$ on the $\nu - \mu$ limit domain. We observe that such a domain significantly expands in the direction of positive $\nu$ values, as the flange width and the lateral prestress level increase. On the contrary, increasing values of the longitudinal prestrain $\varepsilon_p^*$ produce a contraction of such a domain in the same direction (Fig. 9f).

4. Moment - curvature diagrams

We now examine moment-curvatures diagrams relating the dimensionless bending moment $\mu$ to the dimensionless curvature $\chi\ell_w$ (we let $\chi$ denote the cross-section curvature). Such a study is conducted on accounting for the tensile branch of the stress-strain curve of concrete shown in Fig. 6. We consider the following values of the dimensionless axial force

$$\varepsilon_p^*=[0, \ 0.0026, \ 0.0050]. \quad (25)$$

and different values of the design variables introduced in the previous section (see Figs. 10 and 11). Figs. 10 a-d show the $\mu - \chi\ell_w$ curves of sections with aspect ratio $A/l_w=1/2$ (medium flange I-beam); while Figs. 10 e-h display the analogous curves of sections with $A/l_w=1/3$ (narrow flange I-beam). Figures 11 a-d present the $\mu - \chi\ell_w$ curves of sections with $A/l_w=1$ (wide flange I-beam, or W-beam), for given reinforcement ratios $(A_p/A_c = 0.00375, A_f/A_c = 0.0025)$. Finally, Figs. 11 e-h show the moment – curvature curves of medium flange I-beams with $A_f/A_c = 0.00025$, and different values of the flange reinforcement ratio $A_p/A_c$ (Fig. 8). The results in Figs. 10 and 11 a-d highlight that the cross-section rotation ductility remarkably grows with the lateral prestress level $\sigma_c/f_c$. Such a beneficial effects of the lateral prestress becomes less remarkable in presence of large values of the dimensionless axial force $\nu$. By increasing the flange reinforcement ratio $A_p/A_c$, we observe a slight decrease of the cross-section ductility, which is balanced by an appreciable increase in the ultimate bending capacity (Figs. 11 e-h).
Fig. 9 Design charts of laterally prestressed I-beams for different cross-section geometries, lateral prestress values and longitudinal prestrains.
Fig. 10 Dimensionless bending moment ($\mu$) vs. dimensionless curvature ($\chi l_w$) curves of RC I-beams, for different cross-section geometries, lateral prestress values and axial loads.
Fig. 11 Dimensionless bending moment ($\mu$) vs. dimensionless curvature ($\chi l_w$) curves of RC I-beams, for different cross-section geometries, lateral prestress values; axial loads; and steel reinforcement ratios.
5. Concluding remarks

We have presented a parametric study on the beneficial effects of active lateral and longitudinal prestress states on the ultimate axial load – bending moment interaction diagrams, and moment-curvature curves of RC I-beams. The numerical results presented in Sects. 3 and 4 have shown that the lateral prestress of the flanges of such elements is able to significantly enhance both the ultimate strength and ductility properties of the cross-sections, especially in the case of wide-flange beams. The curvature ductility of laterally prestressed elements gets particularly large in presence of high lateral prestress \( \sigma_l / f_c \); low or moderately low dimensionless axial loads \( \nu = N / A_t f_y \); and weakly reinforcement ratios.

It is worth remarking that the lateral prestress locally enhances the strength and ductility properties of compressed concrete (Sect. 2), and additionally modifies the failure mechanism of the cross-section, by reducing the neutral axis depth at the ultimate limit state (Fig. 8). Furthermore, the improved strength properties of the laterally prestressed concrete lead to an increase in the maximum reinforcing steel area ensuring ductile failure of the cross-section (with the tension steel yielding), as compared to passively confined members.

Overall, we conclude that the amplitude of the lateral prestress can be looked at as a new design variable of seismic resistant RC elements, such as, e.g., structures of RC buildings and bridges in seismically active areas, to be suitably tuned through an optimized design of the zones to be laterally prestressed, and the magnitude of the lateral prestress.

We address the enlargement of the parametric study presented in Sects. 3 and 4 to future work. Another challenging generalization of the current research regards the study of the serviceability limit state on crack width of prestressed RC elements, to be conducted on combining the constitutive assumptions introduced in the present work with variational fracture models [Fraternali, 2007; Schmidt et al., 2009; Fraternali et al., 2010]. Additional future extensions of the present study might regard the mechanics of sandwich structures incorporating prestressed layers [Mortazavi et al., 2003; Mukherjee and Rai, 2009; El Sayed et al., 2009], as well as the use of tensegrity networks and strut-and-tie models for the optimal design of spatially precompressed RC and masonry structures [Fraternali et al., 2011; Fraternali et al., 2002; Fraternali, 2010; Fraternali, 2011; Fraternali et al., 2012; Skelton et al., 2013].

6. Acknowledgments

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7. References


