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RESEARCH PAPER

A minimal mass deployable structure for solar energy harvesting on water canals

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Abstract This paper produces a design for a minimal mass, deployable support structure for a solar panel covering of water canals. The results are based upon the minimal mass properties of tensegrity structures. The efficient structure is a tensegrity system which has an optimal *complexity* (i.e. an optimal number of members) for minimal mass. This optimal complexity is derived in this paper, along with deployable schemes which are useful for construction, repairs, for Sun following, and for servicing. It is shown that the minimal structure naturally has deployable features so that extra mass is not needed to add the multifunctional features. The design of bridge structures with tensegrity architecture will show an optimal complexity depending only on material choices and external loads. The minimization problem considers a distributed load (from weight of solar panels and wind loads), subject to buckling and yielding constraints. The result is shown to be a Class 1 Tensegrity *substructure* (support structure only below the deck). These structures, composed of axially-loaded members (tension and compressive elements), can be easily deployable and have many port-able applications for small spans. The focus of this paper is an application of these minimal mass tensegrity concepts to design shading devices to prevent or reduce evaporation loss, while generating electric power with solar panels as the cover. While the economics of the proposed designs are far from finalized, this paper shows a technical solution that uses the smallest material resources, and shows the technical feasibility of the concept.

Keywords Tensegrity structures · Form-finding · Minimum mass · Deployable structures · Shading façades · Water canals · Solar panels

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1 Introduction

In civil engineering, many different kinds of bridge structures are known and used, such as, e.g.: beam bridges, truss bridges, cantilever bridges, arch bridges, tied arch bridges, suspension bridges, and cable stayed bridges, to name just few examples. Unfortunately, available bridge designs most often do not account for topology optimization and minimal mass.

To stop the evaporation losses, reports have shown (Kahn and Longcore 2014) the economic benefits of covering the aqueducts that bring water to California from the Colorado River. It is also logical that the chosen cover could be solar panels to generate energy without requiring new land, (contrary to the requirements of wind turbines or large solar farms) (Mahurkar 2012). The Narmanda Canal in Gujarat India is an example of solar panel covered aqueduct built in 2012 (Mahurkar 2012). To determine the true achievable benefits of such a concept one must engineer the support system to use the smallest amount of material possible, and then reconsider the economic projections. This paper provides the minimal mass solution to the solar panel support structure.

Tensegrity structures are very efficient, and tend to provide minimal mass solutions to structure design under certain conditions. We propose a tensegrity bridge design

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that has minimal mass among all possible tensegrity topologies (configurations of members). The use of tensegrity architectures to design minimal mass structures has been proposed for tensile structures subject to stiffness constraints (Skelton and Nagase 2012); as well as structures carrying compressive loads (Skelton and de Oliveira 2010a); cantilevered bending loads (Skelton and de Oliveira 2010b); torsional loads (Skelton and de Oliveira 2010c), simply - supported bending loads (Carpentieri et al. 2015); and distributed loads on simply-supported spans, where significant structure is not allowed below the roadway, (Skelton et al. 2014). Of course, minimal mass structures are not new ideas (refer, e.g., to Michell (1904)). A tensegrity design of a pedestrian bridge is given in Rhode-Barbarigos et al. (2010), on using a parametric design approach based on hollow ring modules. A more recent parametric design has been developed by Pichugin et al. (2015), with reference to a bridge structure carrying a uniform vertical load over multiple spans. Interesting studies dealing with structural optimization under compliance and frequency constraints are given in Bochenek and Tajs-Zielinska (2015), Kanno (2013), and Yamada and Kanno (2015), and references therein.

Michell (1904) derived a minimal mass, simply - supported truss structure, subject to yielding constraints. Such a structure is composed of an infinite number of members, and actually consists of a continuum in which the lines of tensile stress and the lines of compressive stress are perpendicular each other. It is worth noting that Michell's truss features a considerably large rise vs span ratio and also yielding is not the actual mode of failure when the number of structural members is finite, rather than infinite. Consider also that practical construction always creates joint mass that calls for a finite number of members in the minimal mass realization of the structure. As a result, minimal design approaches accounting for buckling constraints are required, and will be used in this paper to produce minimal mass configurations of a deployable roof of a water canal. We show that the minimal mass configuration of the analyzed structure features a flat roof (no *superstructure*, only structure below the horizontal), and a very streamlined cross- section, which is also able to tolerate high winds (not examined in the present study). The optimally designed roof structures are light-weight and easily deployable. We refer the reader to Puig et al. (2010) and Tibert and Pellegrino (2002) for a comprehensive review of deployable tensegrity systems, and their application to space structures.

The concept of optimality adopted in this paper regards the achievement of a design that ensures the less use of material under equilibrium constraints for given external loads and under specified failure mechanisms of the members (bars and cables). We deal with the minimum mass design of 3D networks of deployable tensegrity structures carrying vertical loads distributed over the surface of an array of solar panels (solar thermal collectors and/or photovoltaic panels). The aim of the present paper is to design minimal mass structures (i.e. economically convenient) that span any given water canal and carry energy harvesting devices, at the same time.

The examined structures are designed to generate electric power while operating as horizontal shading devices for water canals, which are principally aimed at reducing water evaporation. A deck made of solar panels is supported by a network of truss structures with tensegrity architecture, which are connected each other through a stabilizing network of cables. The overall structure is foldable and deployable and is controlled by stretching or relaxing the transverse cables. A minimum mass design leads to lightweight structures easily deployed and maneuverable to aid construction, assembly, servicing, and repair.

The application of interest in this paper is in any canal which brings water to cities that are long distances from a river. As an example, we compute the design for a 400 miles canal bringing water to San Diego from the Colorado River. The technical goal is two-fold: i) to stop or reduce the evaporative losses in such canals, and ii) to use the space above

Fig. 1 The canal cover built in Narmada, India (Mahurkar 2012), and proposed minimal mass concept (tensile cables are yellow and compressive members are *red*)

the canal to generate power using solar panels. This is not a new idea. The UCLA study by Kahn and Longcore (2014) discusses some of the economic issues related to such goals. The article by Merchant (2014) describes a realization developed in India. The UCLA report suggests environmental improvements and challenges, while the India report (Mahurkar 2012) demonstrates feasibility with systems that have been operational since 2012. We here seek for optimal solutions using minimal material resources, and a deployment strategy that erects a light-weight structure (see Fig. 1).

Our motivation is to reduce engineering and construction costs, proposing a support structure for a solar roof that can be actually employed to cover long canals. The panels will not be exactly flat to allow water runoff. Neither the panels nor the support structure will touch the water.

The remainder of the paper is organized as follows. Section 2 describes the tensegrity bridge with parametric topology to be used in this case and defines the loads to be carried. Section 3 describes deployment schemes of the examined bridge, both for construction and maintenance. The following Section 4 discusses the strategies adopted to get a minimal mass design. Numerical results are provided in Section 5. Conclusions are offered at the end (Section 6).

2 Description of the model

The minimal mass of a cable with loaded length s, yielding strength σ_s, mass density ρ_s, and maximal tension t_s is

$$m_s = \frac{\rho_s}{\sigma_s} t_s s. \tag{1}$$

To avoid buckling, the minimal mass of a circular bar of length b, modulus of elasticity E_b, and maximal force f_b is

$$m_{b,B} = 2\rho_b b^2 \sqrt{\frac{f_b}{\pi E_b}}. \tag{2}$$

In the designs of this paper, we will assume buckling as a mode of failure of compressive members since it has been shown in Carpentieri et al. (2015) that buckling is the mode of failure in most of the practical cases, and indeed, in our design. In particular, minimal mass designs subjected to buckling constraints automatically also satisfy yielding constraints if it results (refer, e.g., to Carpentieri et al. (2015))

$$\frac{f_b}{b^2} < \frac{4\sigma_b^2}{\pi E_b}. \tag{3}$$

2.1 Description of the tensegrity model

The paper Carpentieri et al. (2015) finds a planar tensegrity bridge structure that minimizes the sum of deck mass, structural mass, and joint mass. The topology of such a structure depends on several design parameters, which describe its complexity (i.e. the total number of members) and geometry, and are allowed to change in order to achieve an *optimal complexity (minimal mass design)*.

The solution is a Class 1 tensegrity structure (compressive members do not share common vertices) characterized by a finite number of members. That is, the optimal structure is not a continuum (in contrast to Michell's truss, Michell (1904)) but a discrete structure with an optimal number of elements. This optimal number depends on material choice, the span, and the external load. The bridge has no structure above the horizontal line (we call this a *substructure bridge*). The present study assures that the most efficient structure does not extend above horizontal, making it ideal for our proposed solar array surface, since the surface is horizontal, and does not generate any shadows on the solar panels.

For a water canal application, Fig. 2 shows a 3D deployable flat roof made of repetitive 2D *substructure* bridges with multi-scale topology defined in Fig. 3. Each planar *substructure* bridge (Fig. 3) is constrained with two fixed hinges at both ends with vertical reactions $F_{tot}/2$ and horizontal reactions w_x (in practice these hinges might be pulleys that allow roll-up during construction or repair). As illustrated in Fig. 2, this module can be replicated (along the longitudinal direction) to build a deployable three-dimensional structure able to carry vertical loads distributed on the horizontal plane of the solar array.

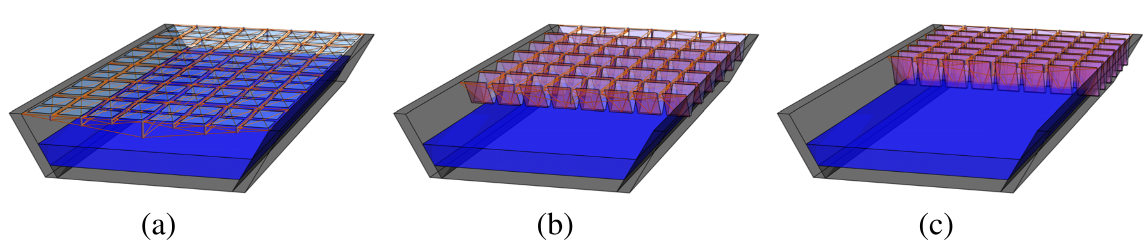


Fig. 2 Different configurations of a deployable solar roof for water canals: **a** open onfiguration, **b** transition between open/closed configurations, **c** closed configuration

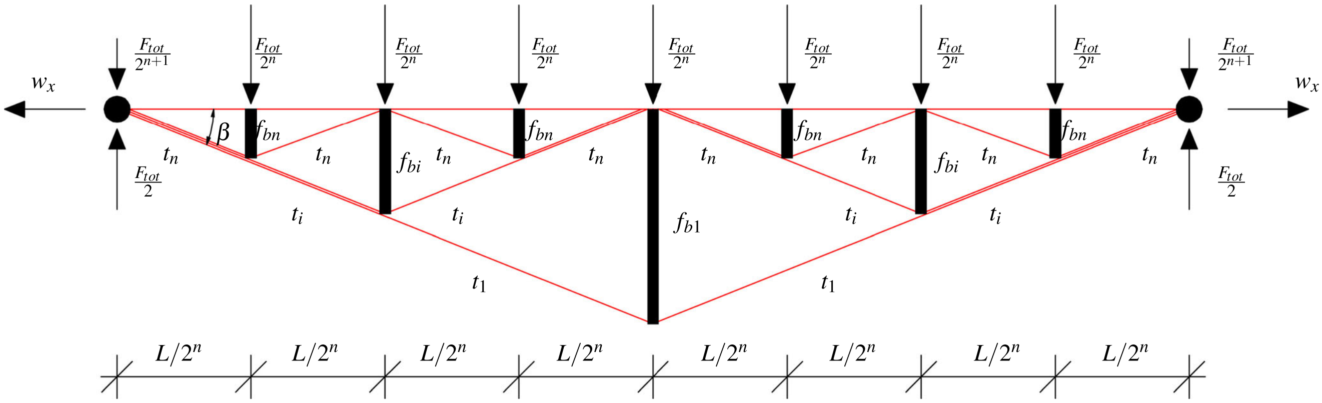


Fig. 3 Adopted notations for forces and lengths of bars and cables for a *substructure* with generic complexity $(n, p, q) = (n, 1, 0)$

3 Description of the deployment scheme

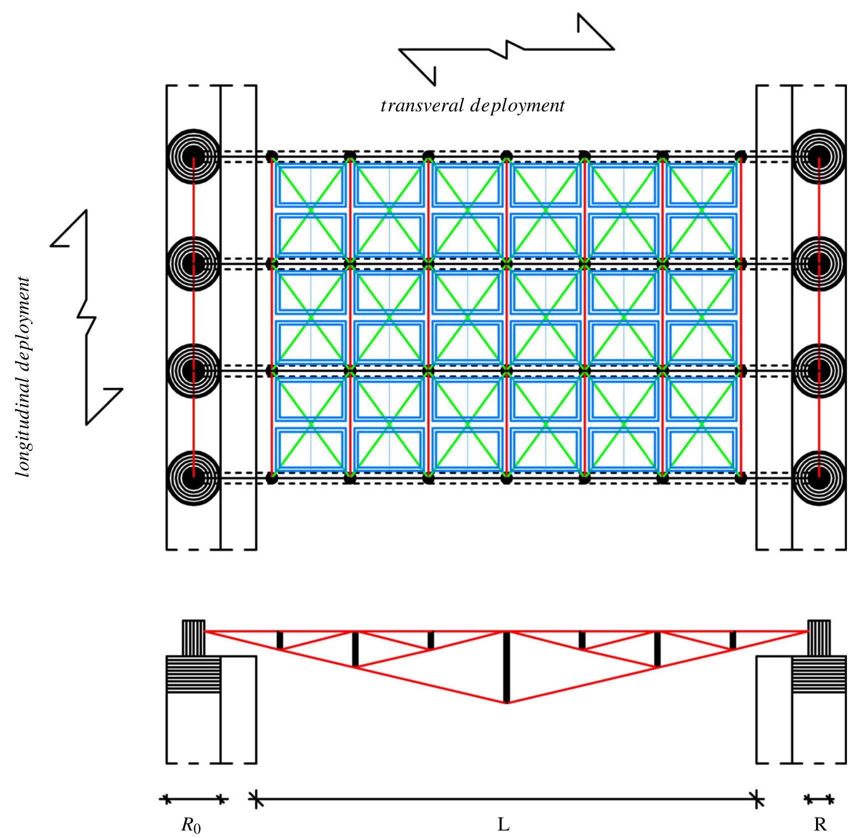
Two different deployment features are incorporated into this design; one for construction (Fig. 4), and one for maintenance (Fig. 2). We will call the motion for construction, *transverse deployment*, described as follows. Since the network is a class 1 tensegrity one can roll up the cables. Imagine the truss system (before the solar panels are installed) rolled up on a large reel of inner radius R_0 and radius after rollup R (see Fig. 4). To compute the required radius of the reel, let L be the cable length required to cross the canal, let r be the cable radius, let v be the number of revolutions required to rollup a cable of length L. The radius of the reel after rollup is $R = R_0 + 2vr$, and the length of the cable rollup is $L = 2\pi \sum_v (R_0 + 2ir) = 2\pi(vR_0 + rv(v + 1))$.

Then one can show that the required radius of the reel is

$$R = R_0 + 2vr = 3R_0 + 2r\left[-1 + \sqrt{1 + \frac{2L}{\pi r\left(1 + \frac{R_0}{r}\right)^2}}\right]. \tag{4}$$

The width of the reel must be greater than the length of the longest bar in the bridge truss network (about 1

Fig. 4 Description of the control system for the deployability



meter for a 20 meters span). One end of the reeled bridge network is secured to the bank foundation (at the reel location) and the other end attached to a cable across the water on the opposite bank. By pulling this cable across the canal the truss network unreels across the canal. In succession, as the truss is pulled across (while maintaining sufficient tension to remain above water level), the solar panels can be installed (attached to the cables) at the canal bank as the cable pulls the network across the canal.

The second type of deployment is perpendicular to the first one, and is called the *longitudinal deployment*, see Fig. 1. This deployment is along the centerline direction of the canal. After damage to a section, the entire width at that location is opened (cable disconnects), this deployment can create an opening of the array to allow access to the water for any reason, such as cleaning, servicing, removing debris from the water, or repairing solar panels.

This longitudinal deploy-ability is assured by controlling the actual aspect angle α_d in Fig. 5. This angle is controlled by a motor that turns a tire on a level concrete track, to roll the bridge sections closer to each other (for servicing or repair), or further apart (for deployment to operational configuration). The angle α_d is a small angle (about 1 deg), determined by the tension selected for the cross cables supporting the panels. Hence the solar panels can face vertical within 1 deg.

The planar bridge topology is considered here to elucidate the fundamental properties that are important in the vertical plane. We use the nomenclature from (Carpentieri et al. 2015), referring to Figs. 3 and 6.

1. A *substructure* bridge has no structure above the deck level.
2. n means the number of self-similar iterations involved in the design.
3. p means the complexity of each iteration in the *substructure*.
4. β is the aspect angle of the *substructure* measured from the horizontal.

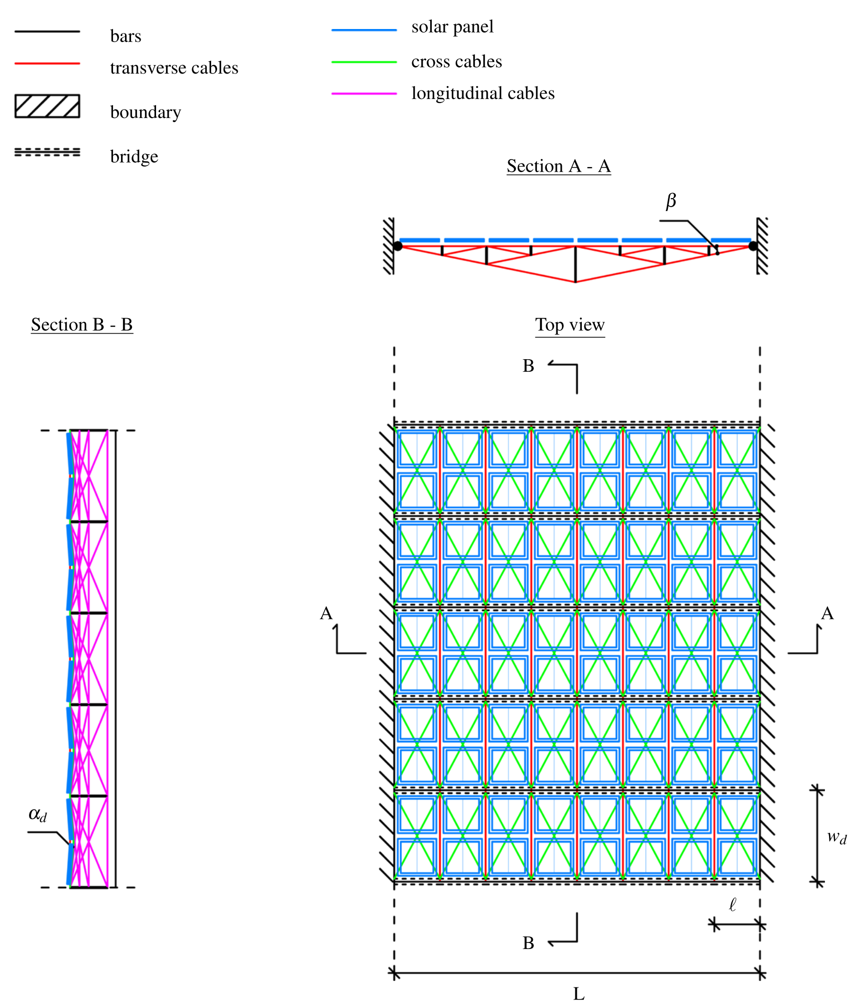


Fig. 5 Schematic of a deployable tensegrity system equipped with solar panels

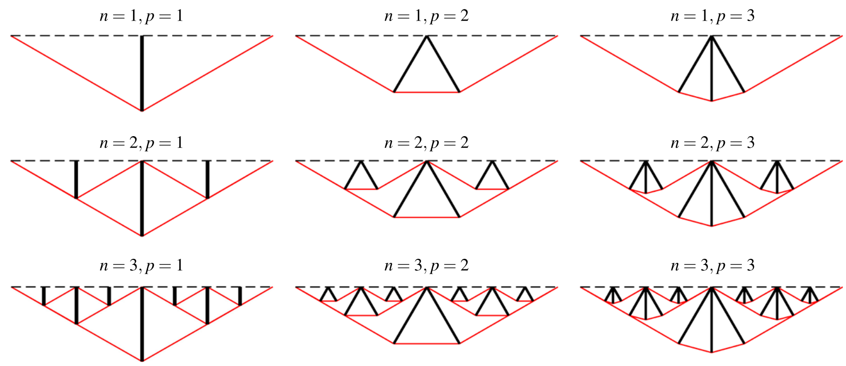


Fig. 6 Exemplary geometries of *substructures* for different values of the complexity parameters n (increasing downward) and p (increasing rightward)

For a tensegrity bridge with generic complexities n and p (see Fig. 3), the total number of nodes n_n of each topology is given by

$$n_n = p\left(2^n - 1\right) + 2^n + 1. \tag{5}$$

For the *substructure* bridge, the number of bars n_b and the number of cables n_s are

$$n_b = p\left(2^n - 1\right), \quad n_s = (p+1)\left(2^n - 1\right) + 2^n. \tag{6}$$

The bridge structures must be stabilized out of the plane with a set of longitudinal cables as illustrated in Fig. 5. In particular longitudinal cables (the magenta elements showed in Section B-B of Fig. 5) are used to prevent out of plane vertical movement.

The deck is composed of different orders of cables (refer to Figs. 5, 7)

1. *longitudinal cables*: the elements connecting each tensegrity bridge unit along the length of the canal;
2. *transverse cables*: the elements of each tensegrity bridge lying on the transversal direction;
3. *cross cables*: the elements that directly carry the solar panel loads and transfer their weight to the bridge structures.

Let F be the total external vertical load for the solar panels to be carried by one planar bridge structure. Each deck section will be loaded by

$$f_p = \frac{F}{2^n}. \tag{7}$$

It will be convenient to define the following constant

$$\eta = \frac{\rho_b L}{(\rho_s/\sigma_s)\sqrt{\pi E_b F}}, \tag{8}$$

and define a normalization of the system mass m by the dimensionless quantity μ

$$\mu = \frac{m}{(\rho_s/\sigma_s)\, F L}. \tag{9}$$

The total vertical force F_{tot} can be computed designing the cross cables represented in Fig. 7. These cables directly support two different solar panel modules of sizes ℓ by

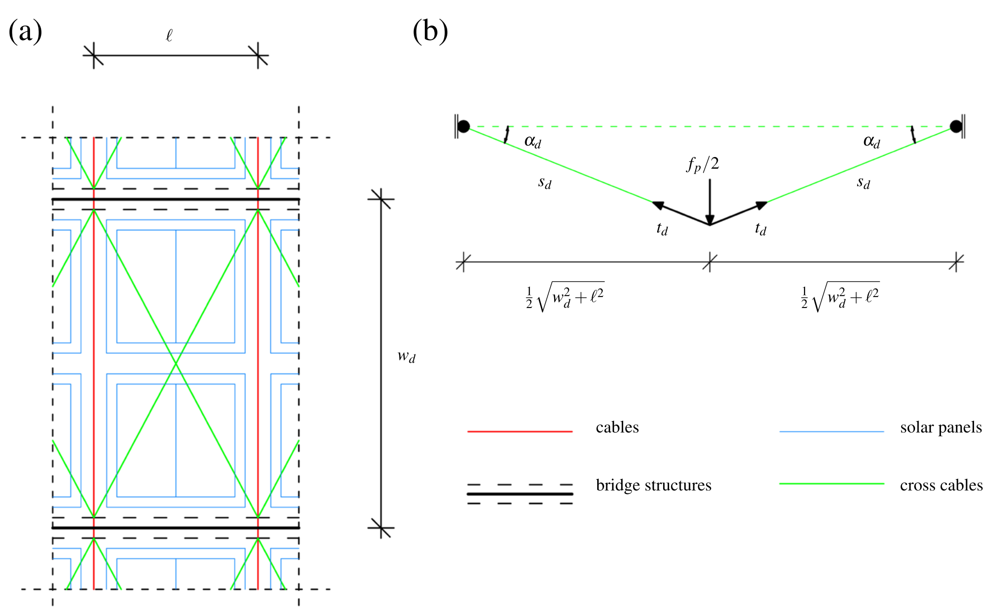


Fig. 7 Details of the canal structure: **a** deck system, **b** deformed shape of the deck cross cables subjected to the solar panel force

$w_d/2$ (see Fig. 7). We design these cables assuming that, at the fully-deployed configuration of the structure, the cross cables are inclined at a fixed angle α_d with respect to the horizontal (Fig. 7). At this configuration the tensile force in each cross cable is

$$t_d = \frac{f_p}{4 \sin \alpha_d}, \tag{10}$$

and the length of each cable is

$$s_d = \frac{\sqrt{w_d^2 + \ell^2}}{2 \cos \alpha_d}. \tag{11}$$

By using (1) we can compute the total mass of the cross cables as

$$m_d = 4\frac{\rho_d}{\sigma_d} t_d s_d = \frac{\rho_d f_p}{\sigma_d} \frac{\sqrt{w_d^2 + \ell^2}}{2 \sin \alpha_d \cos \alpha_d}. \tag{12}$$

Then, the normalized total mass of the deck structure is

$$\mu_d^* = \frac{2^n \, m_d}{(\rho_s/\sigma_s)\, FL}. \tag{13}$$

The total force acting on each internal node on the deck is then the sum of the forces due to the external loads and the force due to the deck load

$$F_{tot} = F + 2^n \, m_d \, g. \tag{14}$$

4 Analytical results

In this section we study the minimal mass of bridges with complexity n. We make use of the notation illustrated in Fig. 3 in which complexity p is fixed to be one. Each iteration $n = 1, 2, \ldots$ generates different lengths of bars and cables. We need not consider $p > 1$ because Carpentieri et al. (2015) have shown that $p = 1$ is the minimal mass solution of a simply-supported *substructure* bridge under buckling constraints. The lengths at the i^{th} iteration are

$$b_i = \frac{L}{2^i} \tan \beta, \qquad s_i = \frac{L}{2^i \cos \beta}, \qquad i = 1 - n. \tag{15}$$

Observing the multi-scale structure of Fig. 3 it's clear that the number of bars and the number of cables at the i^{th} self-similar iteration are

$$n_{si} = 2^i, \quad n_{bi} = 2^{i-1}. \tag{16}$$

In this case the total force applied to the bridge structure is given by (14) and then the forces in each member become

$$f_{bi} = \frac{F + 2^n m_d g}{2^i}, \quad t_{si} = \frac{F + 2^n m_d g}{2^{(1+i)} \sin \beta}. \tag{17}$$

where the mass m_B at the buckling condition is

$$m_B = \frac{2\rho_b}{\sqrt{\pi E_b}} \sum_{i=1}^{n_b} b_i^2 \sqrt{f_{b,i}} + \frac{\rho_s}{\sigma_s} \sum_{i=1}^{n_s} t_i s_i, \tag{18}$$

where (b_i, s_i) is respectively the length of the i^{th} bar or cable, and $(f_{b,i}, t_i)$ is respectively the force in the i^{th} bar or cable.

Consider a *substructure* bridge with topology defined by (5), (6), (15), with complexity n. The minimal mass design under yielding and buckling constraints is given by (Carpentieri et al. 2015)

$$\mu_B^* = \frac{m_B}{(\rho_s/\sigma_s)\, FL} = \beta_1 \frac{(1 + \tan^2 \beta_B^*)}{2 \tan \beta_B^*} + \eta \beta_2 \tan^2 \beta_B^*, \tag{19}$$

where the aspect angle is

$$\beta_B^* = \arctan \left\{ \frac{1}{12\beta_2 \eta} \left[\beta_3 + \beta_1 \left(\frac{\beta_1}{\beta_3} - 1 \right) \right] \right\}, \tag{20}$$

and the coefficients β_i are

$$\beta_1 = \left(1 - \frac{1}{2^n}\right) \left(1 + 2^n g \frac{m_d}{F}\right), \tag{21}$$

$$\beta_2 = \left(\frac{1 + 2\sqrt{2}}{7}\right) \left(1 - \frac{1}{2^{3n/2}}\right) \sqrt{1 + 2^n g \frac{m_d}{F}}, \tag{22}$$

$$\beta_3 = \left(216\beta_1\beta_2^2\eta^2 - \beta_1^3 + 12\sqrt{324\beta_1^2\beta_2^4\eta^4 - 3\beta_1^4\beta_2^2\eta^2}\right)^{1/3}. \tag{23}$$

The minimal mass solution under buckling constraints depends on the material choice for the structural component (bars, cables and deck), on the external force F and span L. If the deck mass m_d is zero then the minimal mass is for complexity $n = 1$. Instead, if F_{tot} is variable with the complexity n through the deck mass m_d (as defined in (14)), the global optimum can be reached for a generic finite complexity n, as a function of the ratio between the total deck force ($2^n \, m_d \, g$) and the total external force (F).

The final total mass to be optimized is then the summation of the mass of the bridge structure (19), the total mass of the deck (13) and the mass of the joints (Ωn_n)

$$\mu_{tot}^* = \mu_B^* + \mu_d^* + \Omega n_n, \tag{24}$$

Table 1 Material properties

	steel	*Spectra®* - *UHMWPE*
ρ $[kg/m^3]$	7862	970
σ $[N/m^2]$	$6.9x10^8$	$2.7x10^9$
E $[N/m^2]$	$2.06x10^{11}$	$120x10^9$

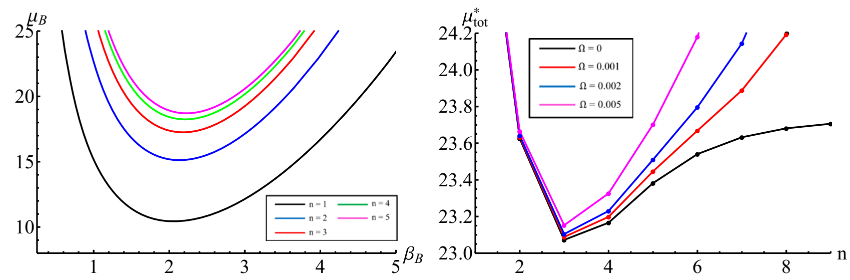


Fig. 8 *Left*: dimensionless masses μ_B vs. aspect angle β_B; *right*: dimensionless total masses μ_{tot} (24) vs. complexity n ($\eta = 7569.04$)

Ω being a factor equal to zero for perfect joints and greater then zero for crudely constructed (cheaper) joints.

5 Numerical results

5.1 Minimal mass roofs of water canals

Let us now focus our attention on numerical results regarding the optimal design of real-life roof structures of water canals featuring different complexities n. The examined structures show the following design data: $L = 30.48\ m$, $F = 12\ kN$, $w_d = 4.88\ m$, $\alpha_d = 1\ deg$, and the material properties in Table 1. We investigate on the optimal values of the following parameters: μ^*_B, μ^*_{tot} and β^*_B, which respectively denote the dimensionless minimal mass of a single bridge unit; the dimensionless minimal mass of the overall system formed by the bridge and the deck; and the optimal aspect angle of the bridge structure, under combined yielding and buckling constraints. The optimal angle β^*_B of the examined structures can be computed from (20), and/or the plots in Fig. 8. It is easy to verify that the minimal mass structure shows a markedly streamlined profile with $\beta^*_B = 2.18\ deg$ (Fig. 8-left). The global minimum of μ^*_{tot} is attained in correspondence with the complexity $n^* = 3$, both for $\Omega = 0$ ($\mu^*_{tot} \cong 23.07$; $m^*_{tot} \cong 3.0318\ kg$, cf. Table 2), and for $\Omega > 0$ (Fig. 8-right). The optimal design leads to 0.10 kg mass per cables and 0.02 kg mass per bars per meter of the canal lengthwise span (cf. Table 2).

Table 2 Optimal masses μ^*_B (19) and μ^*_{tot} (24) and optimal aspect angles β^*_B (20) of *substructure* bridges with steel bars and Spectra® cables, under combined yielding and buckling constraints (B), for different complexities n

n	p	F_{tot} $[N]$	β^*_B $[deg]$	μ^*_B	μ^*_{tot}	m^*_{tot} $[kg]$
1	1	12019.39	2.06	10.4357	25.4791	3.3480
2	1	12010.97	2.13	15.1186	23.6251	3.1044
3	1	12007.50	2.18	17.2522	23.0724	3.0318
4	1	12006.35	2.21	18.2414	23.1662	3.0441
5	1	12006.03	2.23	18.7080	23.3822	3.0725

5.2 Eigenmodes and eigenvectors of the stiffness matrix

Let us examine the stiffness matrix of the planar structure in Fig. 5, for $n = n^* = 3$, $\beta = \beta^*_B = 2.18\ deg$, and different values of the mass m_t of the horizontal cables placed at the deck level (cf. Section A-A of Fig. 5). We remark that $m_t = 0$ corresponds to the minimal mass configuration shown in Section 5.1. We computed the above matrix through the procedure described in Sect. 8 of (Nagase and Skelton 2014). All eigenvalues of the stiffness matrix are positive, which guarantees the global stability of the analyzed structure in the transverse plane (Skelton and de Oliveira 2010c).

The results in Fig. 9 show the first five eigenvalues (sorted in increasing order). In particular, we observe in Fig. 9 that the eigenvalues monotonically increase in magnitude with prestress (hence mass m_t), implying that one can actually improve the overall stability of the structure by slightly increasing prestress, as compared to the minimal mass configuration. Figure 10 shows the first four eigenmodes of the stiffness matrix under examination, for $m_t = 0.2$ kg (tension = 146.12 kN). As we already noticed, the analyzed bridge structure is stabilized out of the plane through the longitudinal cables shown in Fig. 5.

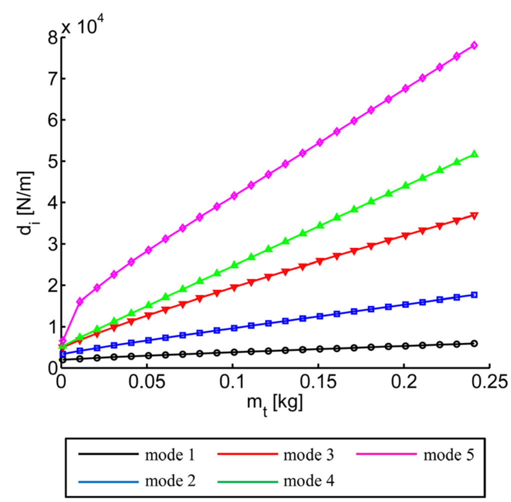


Fig. 9 First five eigenvalues of the stiffness matrix of the examined structure for different values of m_t

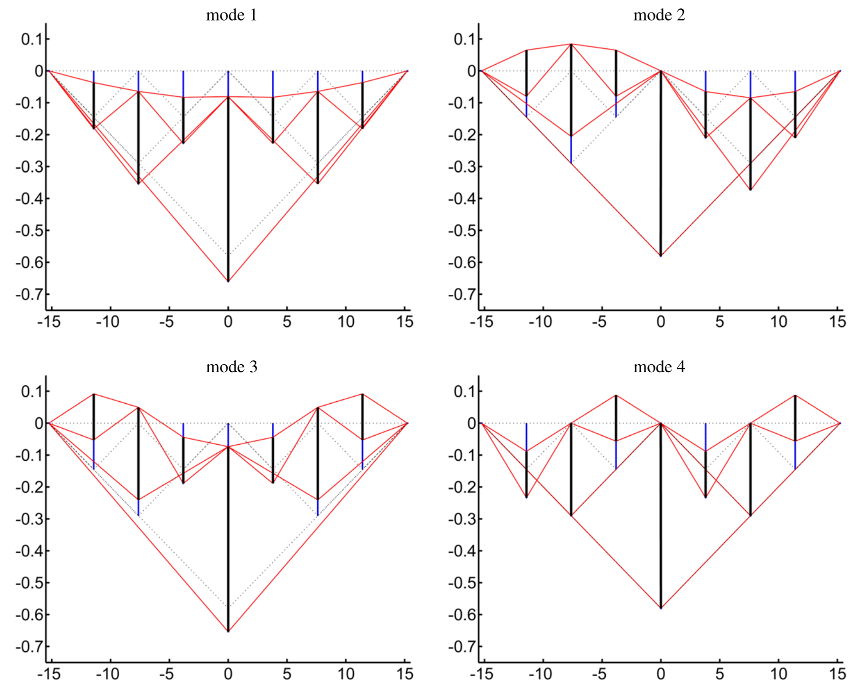


Fig. 10 First four eigenmodes of the stiffness matrix of the examined structure for $m_t = 0.2\,kg$. The blue segments indicate the displacement vectors of the nodes from the undeformed shape (dotted lines). The plots of the eigenmodes make use of different scales for the vertical and horizontal axes, in order to magnify the nodal displacements. For such a reason, the undeformed geometries appear distorted with respect to the real profiles ($\beta = \beta_B^* = 2.18deg$)

6 Concluding remarks

This paper provides closed form solutions (analytical expressions) for minimal mass tensegrity bridge designs to be used as deployable roofs for water canals. The forces, locations, and number of members are optimized to minimize mass subject to buckling (for bars) and yielding (for cables) constraints for a planar structure with fixed-hinge/fixed-hinge boundary conditions.

We have examined a 3D deployable tensegrity structure made of repetitive planar *substructure* bridges (spanning the canal in the transverse direction) conveniently stabilized out of plane with a set of cables, in both the transverse and the longitudinal direction of the canal. Each planar structure has a self-similar type of topology generated by the complexity parameter n. The minimal mass solution yields complexity n^* which depends upon material properties. Moreover, the topology of the 3D structure is function of canal width (L), aspect angle (β) of the *substructures* bridges, longitudinal aspect angle (α_d) governing the deploy-ability of the structure, the distance between consecutive repetitive structures in the longitudinal direction (w_d).

Using steel bars and Spectra®cables, we obtained an optimal structure that shows a very small rise, i.e., a streamlined profile. The given design occupies the minimum volume and mass for the attempt at energy production and shading over water canals Formulas are given which will allow economic tradeoffs between material costs of the structure, the labor cost (assuming price per joint is inversely proportional to mass of the joint) of making more refined joints, and the choice of material (steel, Spectra®, or other). Implicit in these tradeoffs, the optimized complexity n^* of the structure is derived to allow economic decisions on the number of components (bars and cables) that will minimize mass for the given choice of material and joint costs.

Numerical and experimental studies on the dynamics of these structures will follow in subsequent work to impose further design constraints on stiffness issues (vibrational frequencies, mode shapes, displacements for high winds conditions, etc., cf., e.g., (Carpentieri et al. 2015; Modano et al. 2015)), but the capability of all these choices and adjustments are within the free parameters of the designs in this paper. The subsequent dynamics approach will evaluate the value (economics and performance tradeoffs) the use of feedback control for the deployable and service functions, or to adjust the stiffness of the structure (varying the prestress of the cables) to modify stiffness or damping after storm damage. Additional future research lines include the design of different deployment schemes, on examining both straight and curved canals.

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