Leveraging research in computational mechanics to improve undergraduate teaching

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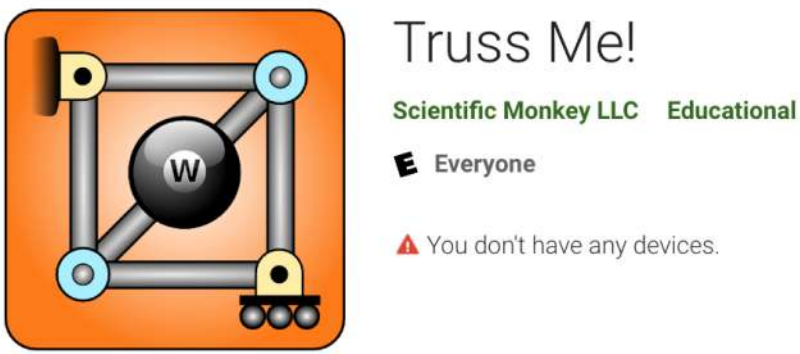
Virtual Seminar
April 20, 2020

Before we start:

- Download Truss Me! (freely available)

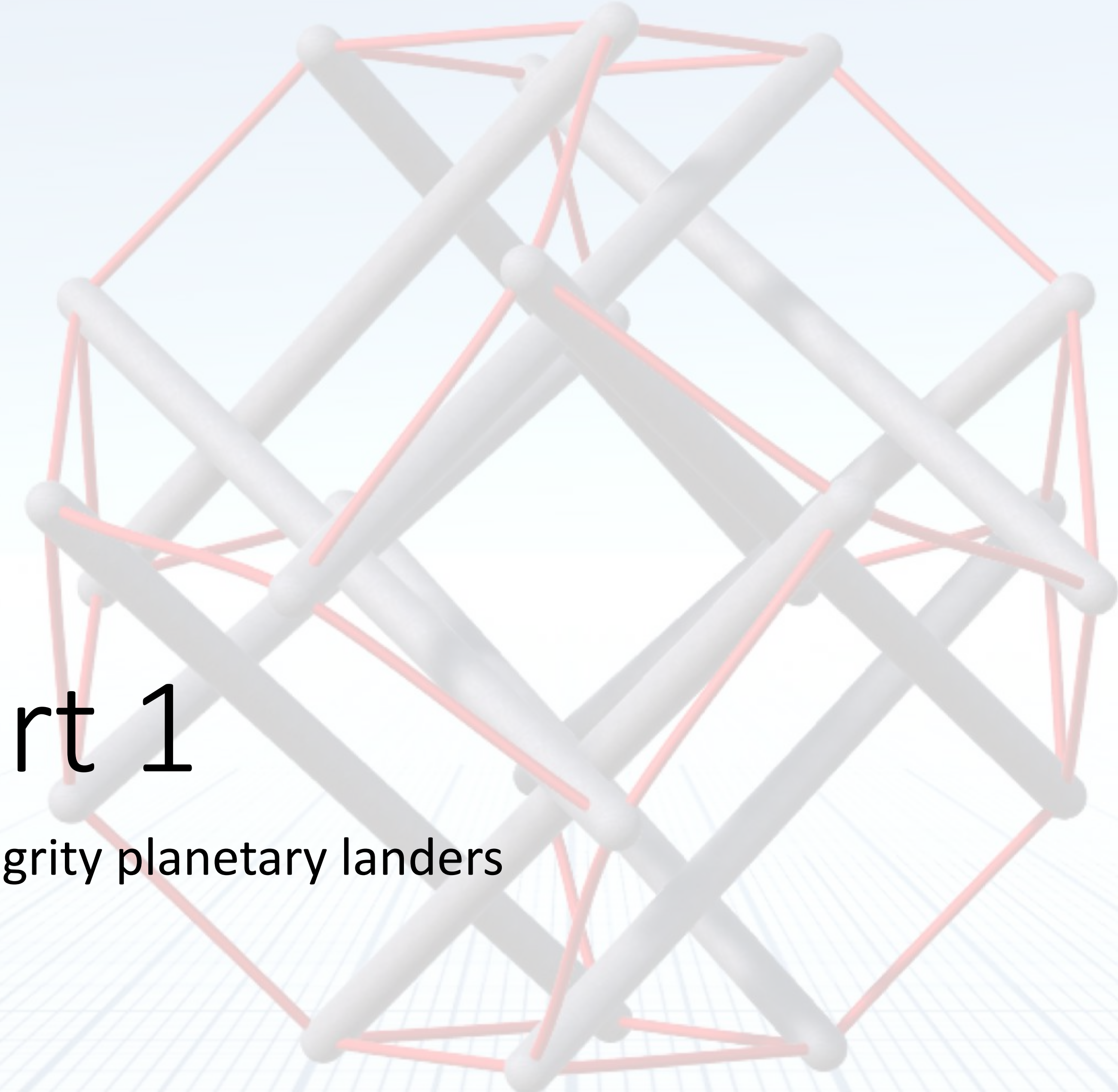


Google Play

Apple's App Store

Outline

- Part 1: Tensegrity structures for planetary landers.
  - Motivation
  - Model development
  - Impact behavior
- Part 2: Truss Me! A game-based learning tool for structural mechanics.
  - Introduction
  - Hands-on activities



Part 1

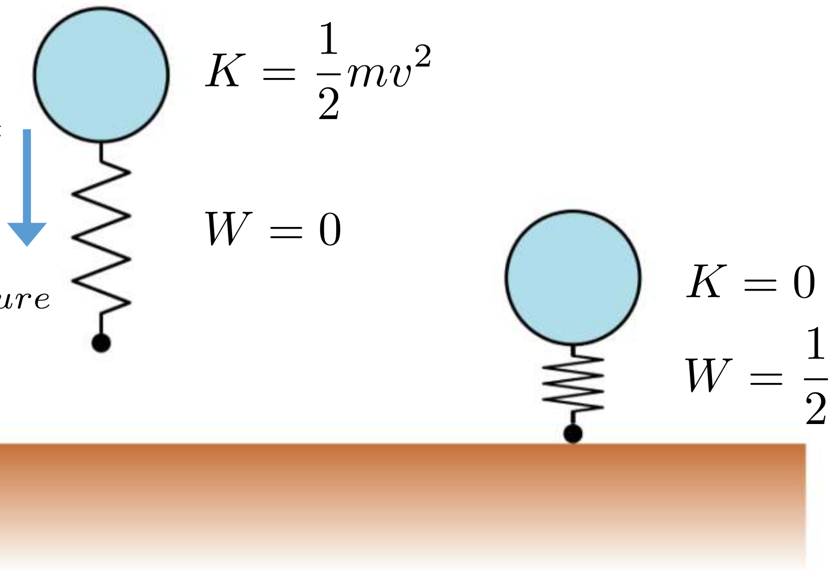
Tensegrity planetary landers

Planetary landing structures 101

$$m = m_{st} + m_{pl}$$

$$v = v_{impact}$$

$$k = k_{structure}$$



$$K = \frac{1}{2}mv^2$$

$$W = 0$$

$$K = 0$$

$$W = \frac{1}{2}k\delta^2$$

Worst-case scenario: impact without dissipation $\Longrightarrow \frac{1}{2}mv^2 = \frac{1}{2}k\delta^2$

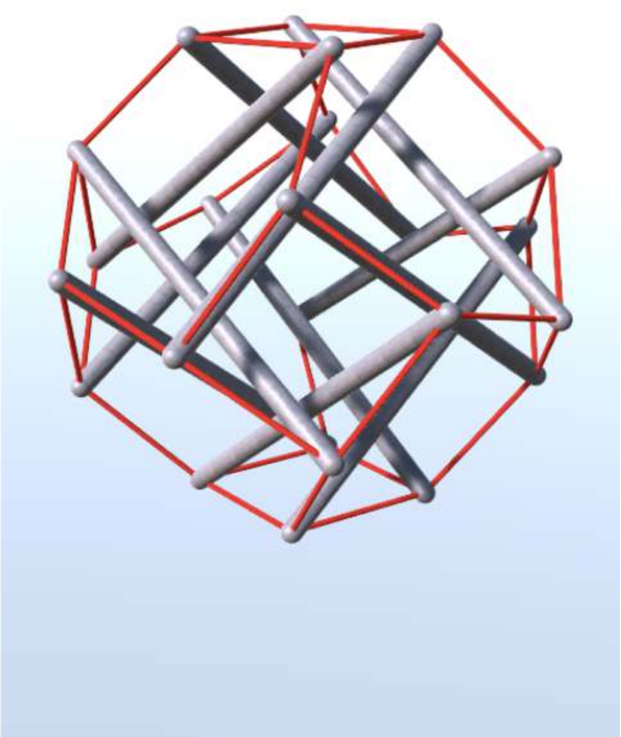
Background

- SunSpiral first proposed the use of tensegrity structures for planetary landers [1].
- Solution inspired by tensegrity toy's ability to recover from extreme deformation.
- Main idea:
    - Structure composed of **rigid bars and elastic cables**
    - Generate locomotion by actuating cable members
- Main limitation:
    - Assumed rigid bars, and all energy absorbed by cables
    - This would lead to mass that does not contribute to energy storage!

[1] V. SunSpiral et al, International Journal of Planetary Probes, 2013

Tensegrity Planetary Lander: our concept

- Design a tensegrity structure that exploits **elasticity of bars**.
- Tensegrity octahedron's super-stable property [1] makes it a good candidate.
- Payload located at the center (cable suspended)
- Main advantages:
  - Higher impact velocity (or higher payload)
  - **Strain energy evenly distributed throughout the structure.**

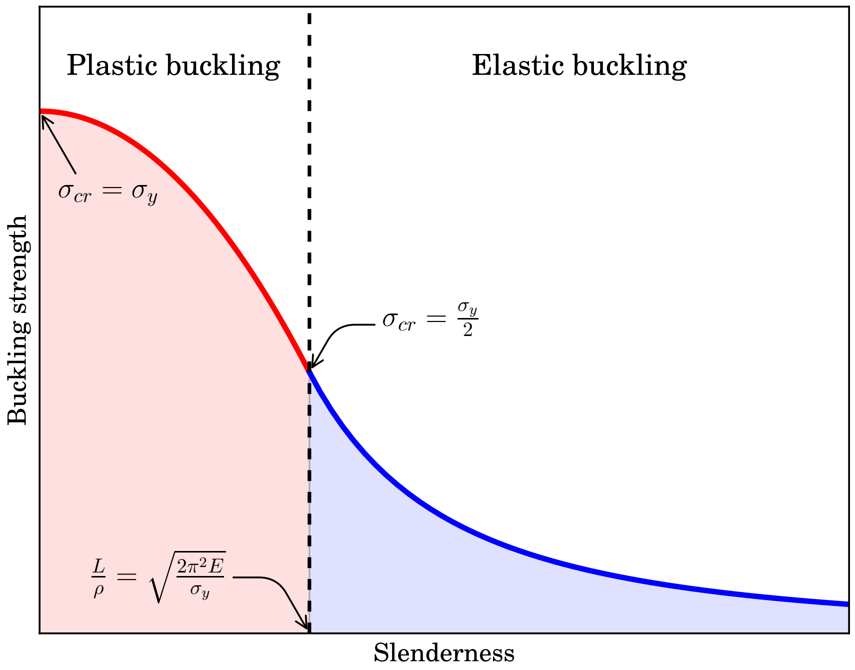


[1] Zhang et al, International Journal of Solids and Structures, 2019

A distinctive design approach:
Let it buckle

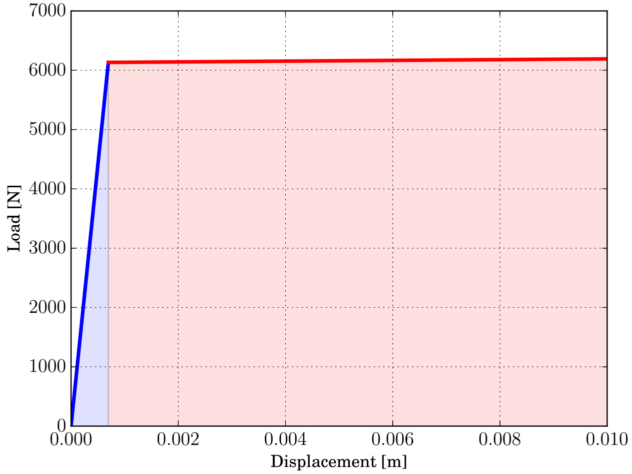
- We propose to design tensegrity structures whose bar members are allowed to buckle in the elastic regime.
- This will bring several benefits (to be discussed later in this presentation) for impact problems.
- From a design perspective, a bar can be easily dimensioned to ensure elastic buckling:

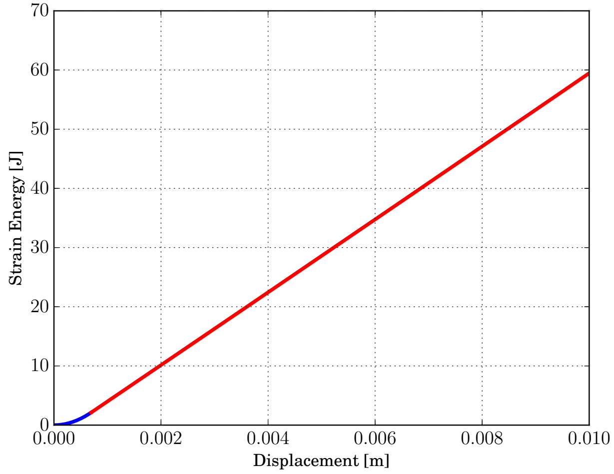
$$\frac{L}{\rho} \geq \sqrt{\frac{2\pi^2 E}{\sigma_y}}$$



Post-buckling behavior

- When loading a simply supported bar axially, the response is linear up to the buckling point.
- Consequently, the stored strain energy is quadratic. This is the amount of energy we would be able to store if buckling is not allowed.
- Beyond buckling, the load does not drop. On the contrary, it remains almost constant (slightly increases)
- Since relatively large displacements can be applied after buckling, large amounts of additional strain energy can be stored on the bar.
- In this example, a 58cm long Ti tubular bar with 19mm diameter and 1mm thickness can store 58.5 J in the post-buckling regime vs 2.1 J if buckling is not allowed (27x increase in this case)





Modeling tensegrity structures

Contents lists available at ScienceDirect

Mechanics of Materials

journal homepage: www.elsevier.com/locate/mechmat

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MECHANICS OF MATERIALS

A reduced-order model for the dynamic and post-buckling behavior of tensegrity structures

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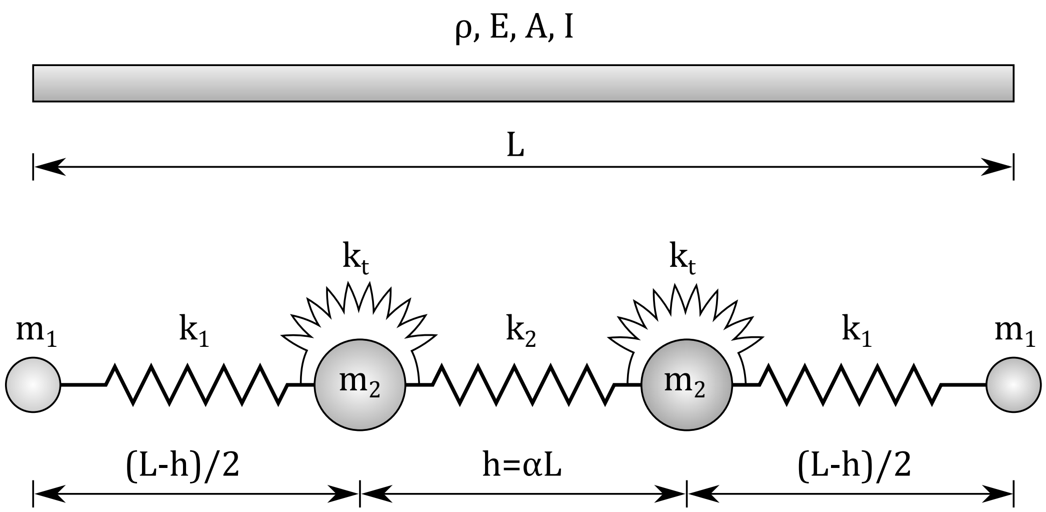
Modeling tensegrity structures



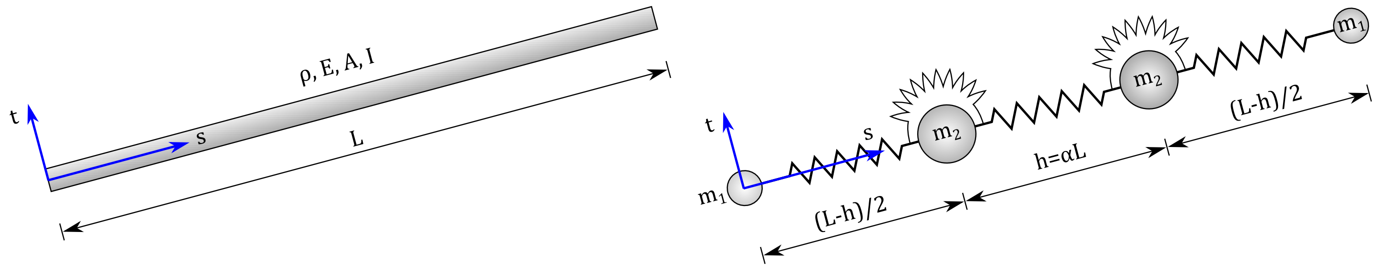
$$k_1 = \frac{2EA}{L-h} = \frac{2EA}{L(1-\alpha)}$$

$$k_2 = \frac{EA}{h} = \frac{EA}{\alpha L}$$

- The continuum bar is discretized as a symmetric system with 4 masses, 3 axial springs and 2 angular springs.
- We must determine all stiffnesses and masses to reproduce required bar properties.
- Computation of stiffness of axial springs is trivial.
- The distance between the two interior masses is left as a discretization parameter ($h=\alpha L$)

Rimoli, MOMS, 2018

Mass properties



- By forcing the discrete system to have the same mass and mass moment of inertia as the continuum bar we obtain:

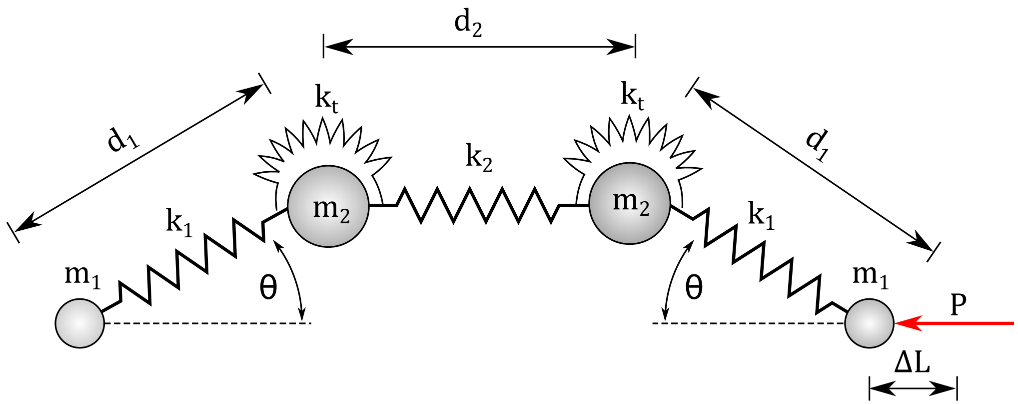
$$
\left.\begin{aligned}
&2m_1 + 2m_2 = \rho A L \\
&\frac{1}{2}m_1 L^2 + \frac{1}{2}m_2\alpha^2 L^2 = \frac{1}{12}\rho A L^3
\end{aligned}\right\}
\Rightarrow
\begin{aligned}
m_1 &= \frac{1}{6}\rho A L\left(\frac{1-3\alpha^2}{1-\alpha^2}\right) \\
m_2 &= \frac{1}{3}\rho A L\left(\frac{1}{1-\alpha^2}\right)
\end{aligned}
$$

- To have positive mass, the previous expressions impose a limit on the value of α

$$0 \leq \alpha \leq \sqrt{3}/3$$

Rimoli, MOMS, 2018

Buckling load



- Let us consider the discretized system in the buckled configuration.
- The potential energy of the system is given by

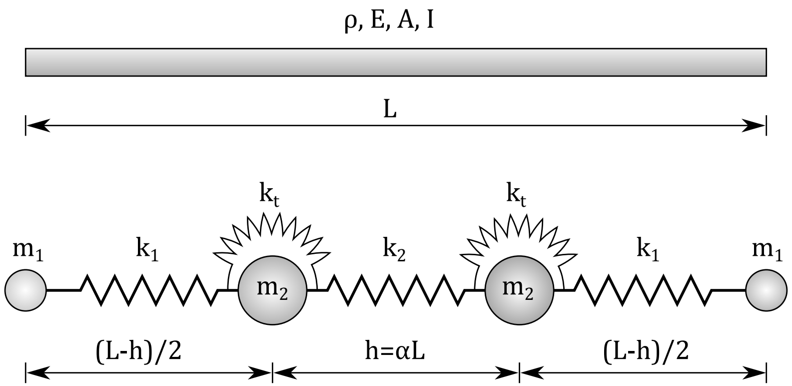
$$\Pi = k_1 \left(d_1 - \frac{L-h}{2} \right)^2 + \frac{k_2}{2} (d_2 - h)^2 + k_t \theta^2 - P\Delta L$$

- Where d_1, d_2, and $\boldsymbol{\theta}$ are the generalized coordinates of the problem, and the displacement at the end of the bar is given by

$$\Delta L = L - 2d_1 \cos(\theta) - d_2$$

- The non-trivial solution gives us: $P_{cr} = \frac{1}{4} k_1 (L-h) - \frac{1}{4}\sqrt{k_1^2 (L-h)^2 - 16 k_1 k_t}$

Buckling load



$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

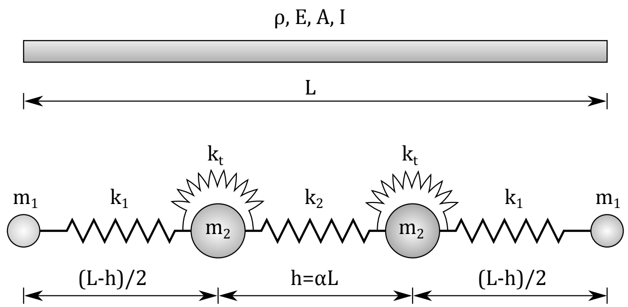
$$P_{cr} = \frac{1}{4}k_1(L-h) - \frac{1}{4}\sqrt{k_1^2(L-h)^2 - 16k_1k_t}$$

- Equating the critical buckling load for the discrete and continuum systems, and solving for k_t we get

$$k_t = \frac{1-\alpha}{2}\frac{\pi^2 EI}{L}\left(1 - \pi^2\frac{I}{AL^2}\right)$$

Rimoli, MOMS, 2018

Discretized system



$$m_1 = \frac{1}{6}\rho A L \left(\frac{1-3\alpha^2}{1-\alpha^2}\right)$$

$$m_2 = \frac{1}{3}\rho A L \left(\frac{1}{1-\alpha^2}\right)$$

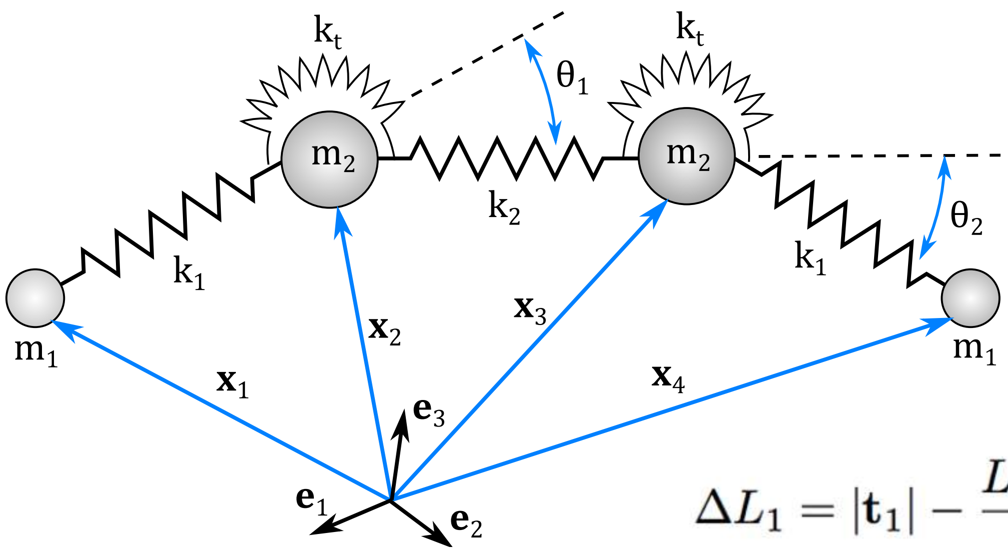
$$k_1 = \frac{2EA}{L(1-\alpha)} \qquad k_2 = \frac{EA}{\alpha L} \qquad k_t = \frac{1-\alpha}{2}\frac{\pi^2 EI}{L}\left(1-\pi^2\frac{I}{AL^2}\right)$$

- The discretized system has the following properties:
  - Same **mass** as continuum bar
  - Same mass **moment of inertia** as the continuum bar
  - Same **axial stiffness** as the continuum bar
  - Same **buckling load** as the continuum bar
  - Nearly same **post-buckling load** as the continuum bar
  - Low error on **first two natural frequencies**, with second frequency exactly twice the first one (with α=1/2)

With only **12 DOFs per** bar we can study dynamic and post-buckling behavior of truss and tensegrity structures/lattices

Rimoli, MOMS, 2018

Implementation details



$$\mathbf{t}_1 = \mathbf{x}_2 - \mathbf{x}_1 \qquad \mathbf{u}_1 = \frac{\mathbf{t}_1}{|\mathbf{t}_1|}$$

$$\mathbf{t}_2 = \mathbf{x}_3 - \mathbf{x}_2 \qquad \mathbf{u}_2 = \frac{\mathbf{t}_2}{|\mathbf{t}_2|}$$

$$\mathbf{t}_3 = \mathbf{x}_4 - \mathbf{x}_3 \qquad \mathbf{u}_3 = \frac{\mathbf{t}_3}{|\mathbf{t}_3|}$$

$$\Delta L_1 = |\mathbf{t}_1| - \frac{L(1-\alpha)}{2}$$

$$\Delta L_2 = |\mathbf{t}_2| - L\alpha$$

$$\Delta L_3 = |\mathbf{t}_3| - \frac{L(1-\alpha)}{2}$$

$$\cos\theta_1 = \mathbf{u}_1 \cdot \mathbf{u}_2$$

$$\cos\theta_2 = \mathbf{u}_2 \cdot \mathbf{u}_3$$

$$U = \frac{1}{2}\left[k_1\left(\Delta L_1^2 + \Delta L_3^2\right) + k_2 \Delta L_2^2 + k_t\left(\theta_1^2 + \theta_2^2\right)\right]$$

$$\mathbf{f}_i = -\frac{\partial U}{\partial \mathbf{x}_i}$$

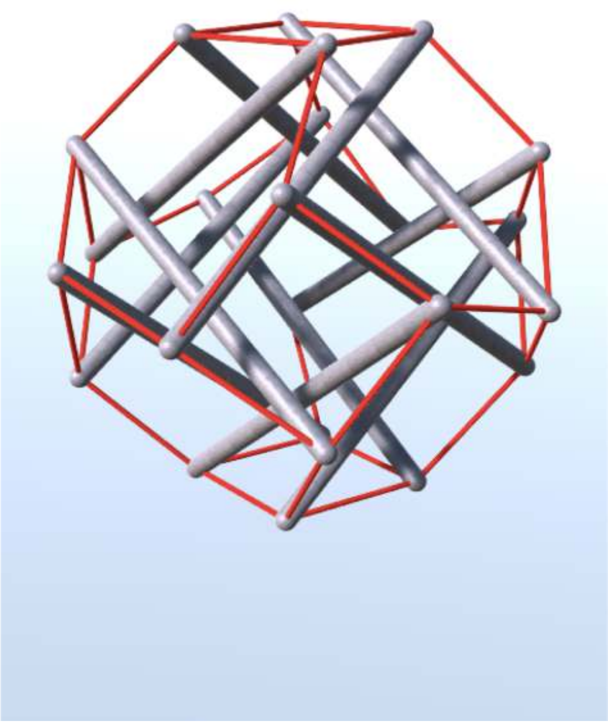
Rimoli, MOMS, 2018

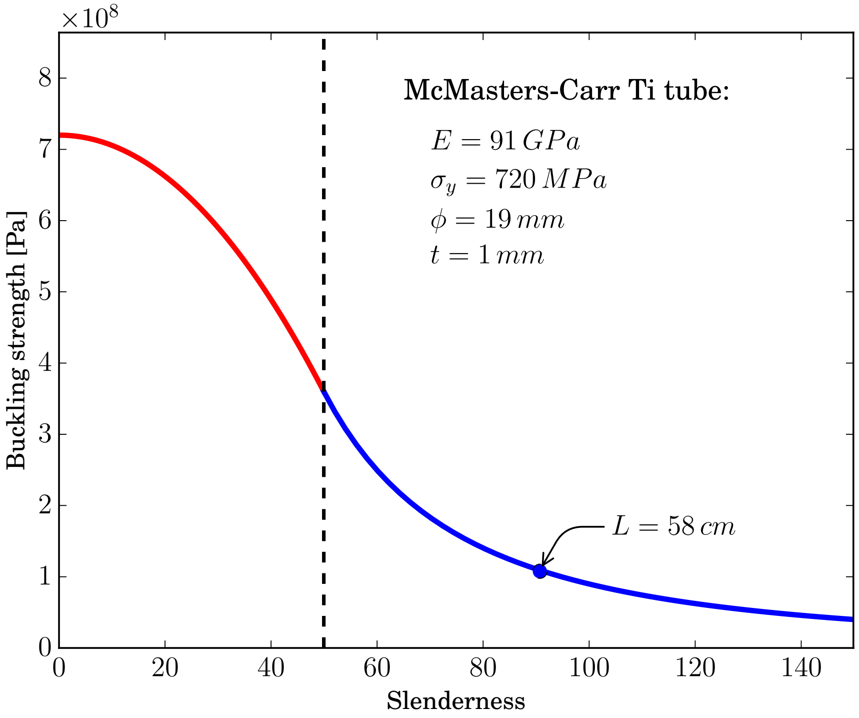
Implementation details

```
// Compute unit vectors
for (int i = 0; i < _n_segments; i++){
    int n1 = _connectivity[i];
    int n2 = _connectivity[i+1];
    _t[i] = points[n2]._position - points[n1]._position;
    _d[i] = _t[i].magnitude;
    _u[i] = _t[i]/_d[i];
}

// Set forces to zero
for(int i=0; i < _n_segments+1; i++){
    F[i].Set(0,0,0);
}

if(elType == "Bar"){
// Compute angular spring forces
    float cost, theta, theta2, theta4, theta6, theta8, coef;
    Vector3 sint, f1, f2, f3;
    for (int i = 0; i< _n_int_points; i++){
        sint = Vector3.Cross(_u[i],_u[i+1]);
        cost = Vector3.Dot(_u[i],_u[i+1]);
        theta = Mathf.Atan(sint.magnitude/cost);
        theta2 = theta * theta;
        theta4 = theta2*theta2;
        theta6 = theta2*theta4;
        theta8 = theta4*theta4;
        coef = _kt[i]/(1f-theta2/6f+theta4/120f-theta6/5040f+theta8/362880f);

        f1 = coef/_d[i]*Vector3.Cross(_u[i],sint);
        f3 = coef/_d[i+1]*Vector3.Cross(_u[i+1],sint);
        f2 = f1+f3;

        F[i] += f1;
        F[i+1] -= f2;
        F[i+2] += f3;
    }
}
```

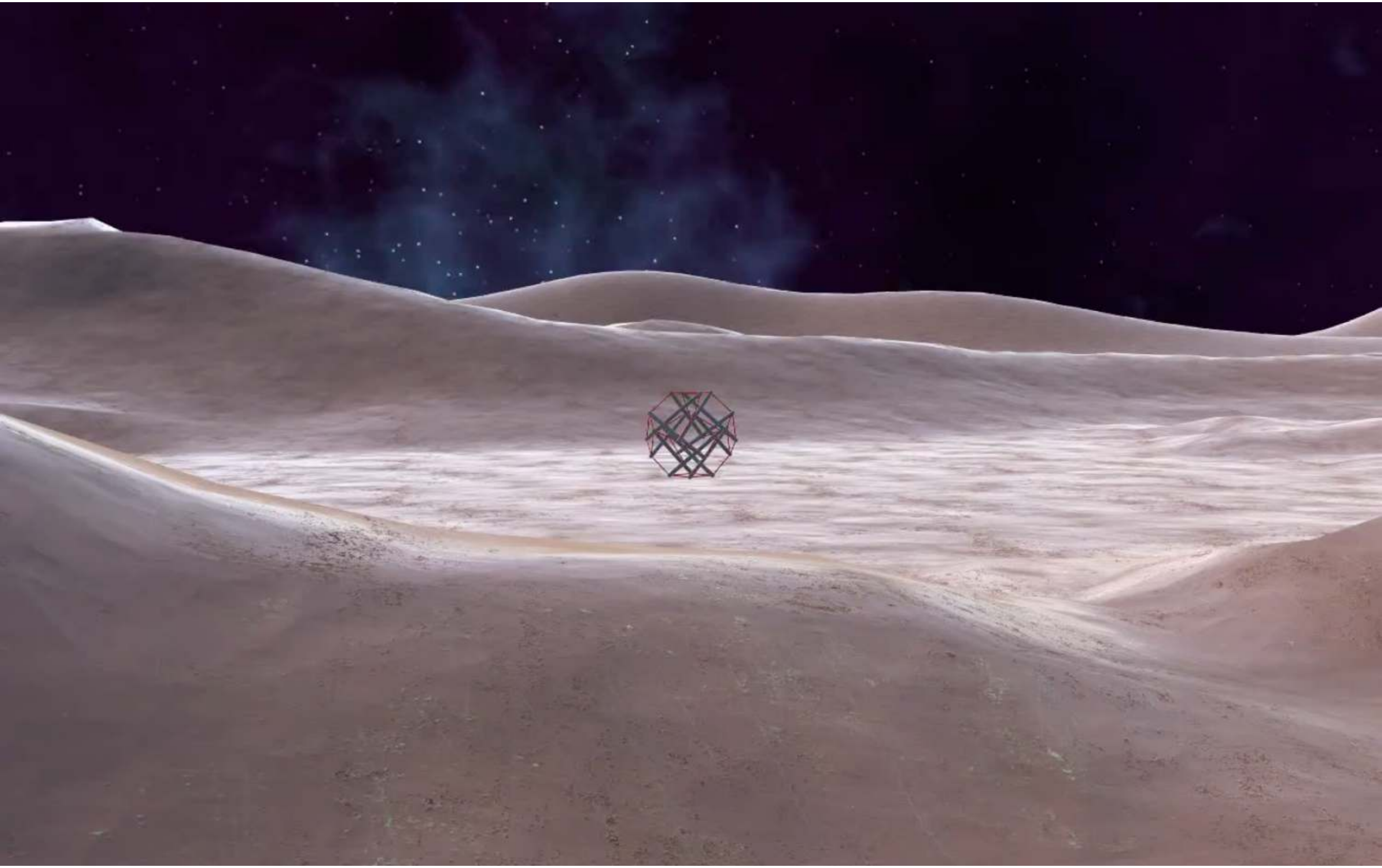
```
// Compute linear spring forces
float force;
for (int i = 0; i < _n_segments ; i++){

    force = _k[i]*(_d[i]-_d0[i]);

    F[i] += force * _u[i];
    F[i+1] -= force * _u[i];

}

// Assemble forces
for(int i=0; i < _n_segments+1; i++){
    points[_connectivity[i]]._force += F[i];
}
```

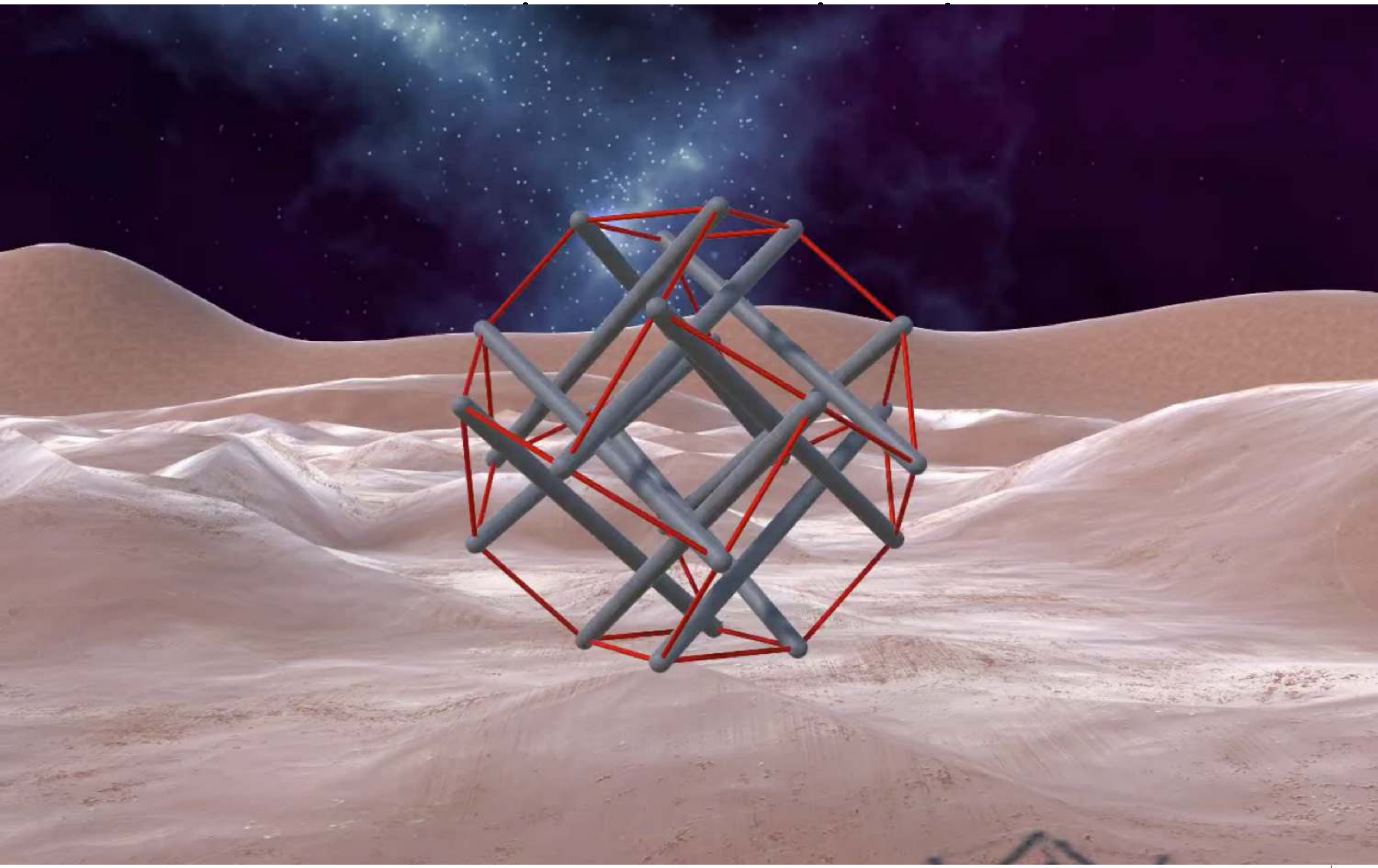
Tensegrity planetary lander: concept design





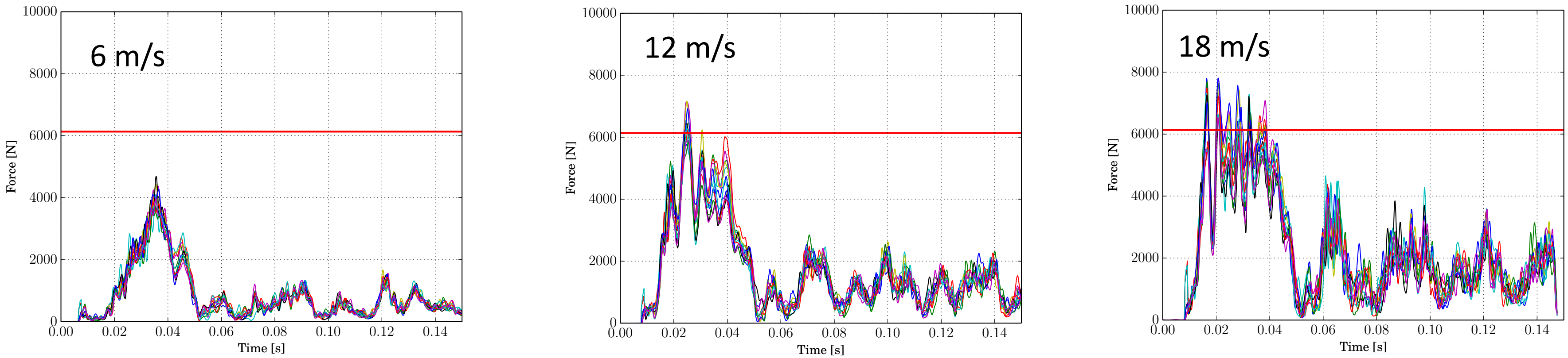
Rimoli, MOMS, 2018



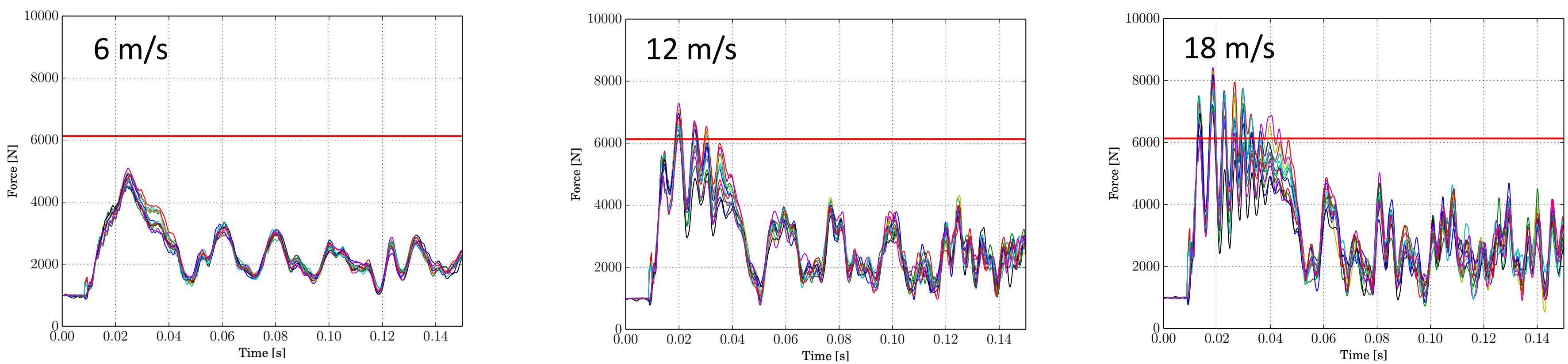


Force history on bars

Without pre-stress

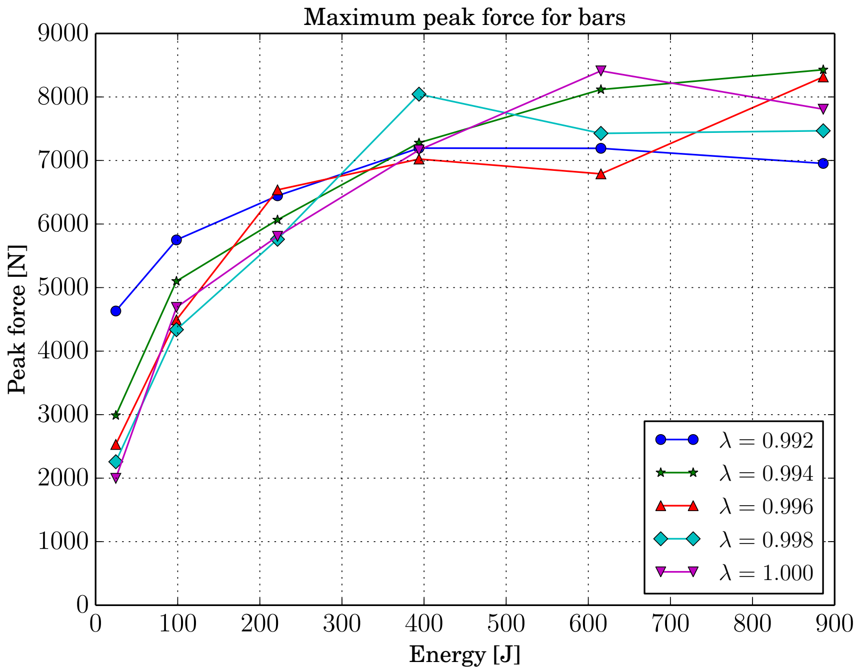


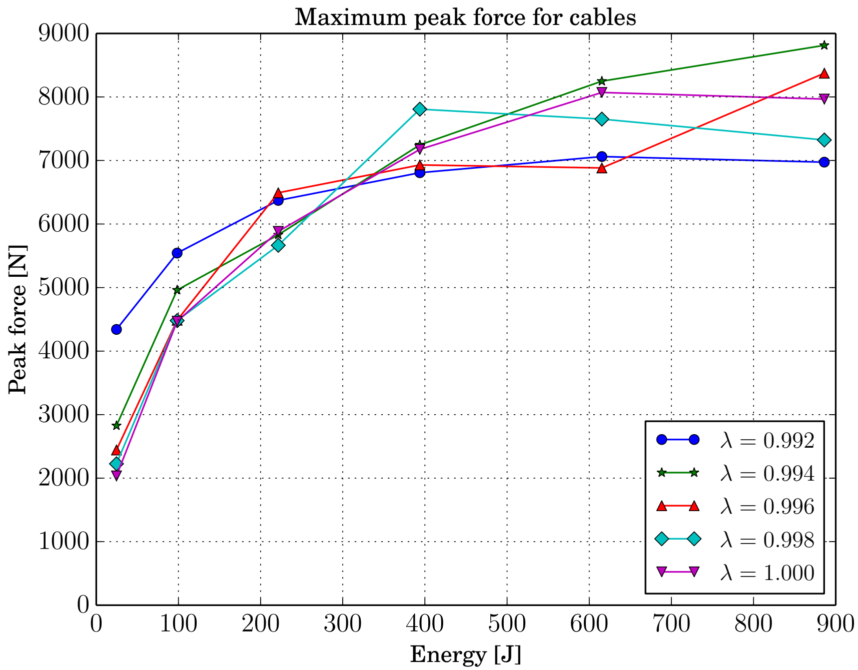
With pre-stress



Rimoli, MOMS, 2018

Maximum peak forces on bars and cables





- By allowing them to buckle, bars act as a load-limiting mechanism.
- It is not only the load that bars experience that is limited: they also limit the maximum load experienced by cables.

Rimoli, MOMS, 2018

3D tensegrity lattice

3D tensegrity lattice

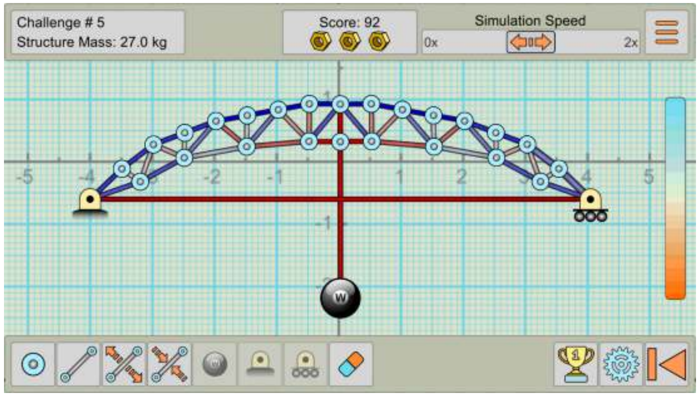
Truss me!

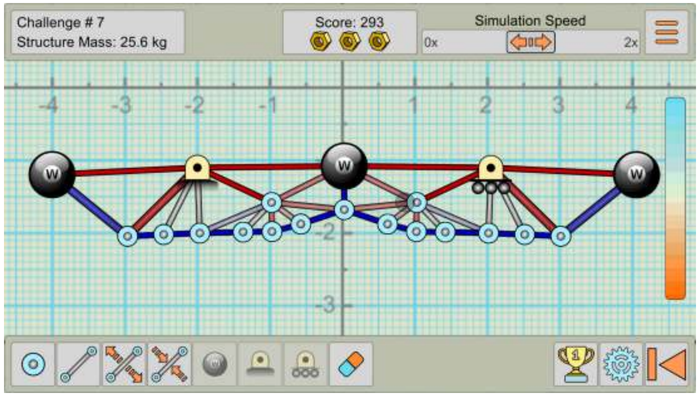
Part 2

A game-based learning tool for structural mechanics

Truss me!: Combining advanced simulations and gaming for engineering education

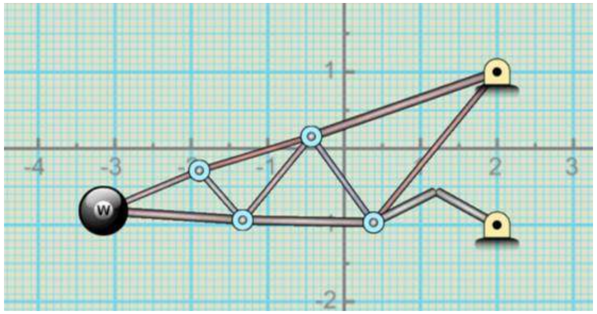
- Designed an educational app to help students **build a conceptual understanding on how truss structures behave and fail.**
- Based on a **state-of-the-art simulation engine** to provide the most realistic experience.
- **Game-like approach**: students solve puzzles, of increasing difficulty, by designing load-bearing structures.
- Available on Google Play and iOS App Store
  - Over 450,000 downloads in 150 countries
- Incorporated for teaching in numerous middle and elementary schools, and some of the most prestigious engineering schools including ETH Zurich, Georgia Tech, Vanderbilt University, Rutgers, and Caltech.



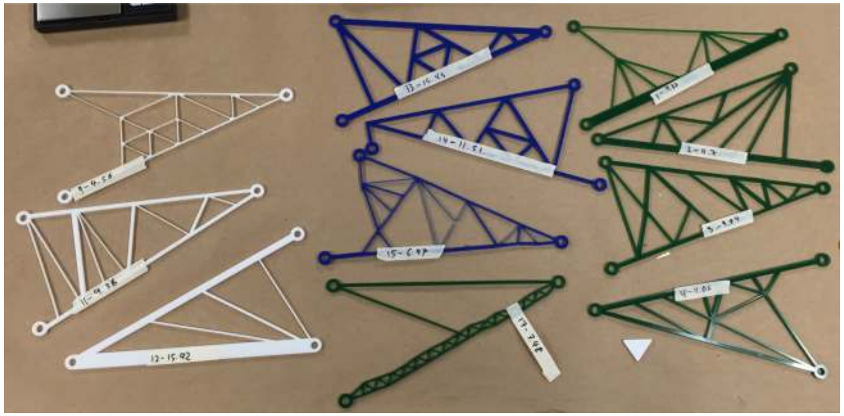


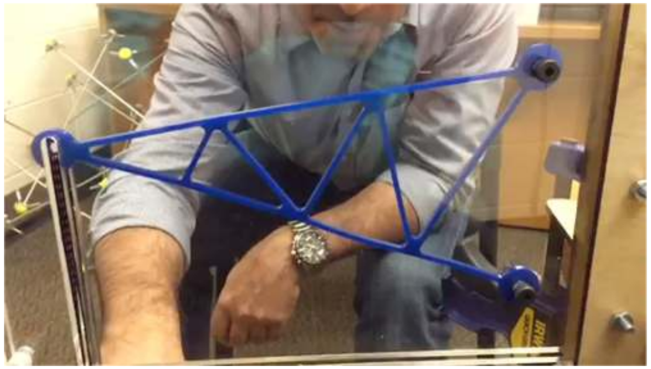
Classroom implementation I:
Simulation of engineering design cycle in the classroom

Design/Analysis



Advanced Manufacturing





Testing

Classroom implementation II:
Semester-long truss design competition

Rules

1- By the end of the day on each dates specified in the calendar at the end of this document, students will upload to canvas a screenshot of their best design for the corresponding challenge.

2- For each challenge, scores from all students will be normalized by the highest score for that level. That is, the top design in the class will get 1 point.

3- By the end of the semester, students will get a final score consisting on the sum of their normalized scores. There is a total of 15 challenges to be solved, so the maximum possible score is 15.

4- The final score of each student will then be normalized by the highest final score in the class. That is, the highest possible normalized score is 1.

5- For each student, the contribution from this competition to the final grade will be 10% multiplied by the normalized final score. That is, the top student in the class will get 10% toward the final grade.

6- The top 4 students in the class will participate on a live finale on Monday, April 22nd 2019.

7- The format of the finale is as follows:

   a. All 4 finalists will get the same challenge to solve, and will be given 15 minutes to get to their best design. All devices will be streamed live to the classroom screen so students in the class will be able to see the design process, and cheer for their favorite competitor if they wish!

   b. The 2 best performers from the first round will pass to the final round, which will have the same format as the previous one.

   c. The winner will be given a $50 gift card, and a $25 gift card will be awarded to the second place. Gift cards are courtesy of Prof. Rimoli ☺

February 2019

Sun	Mon	Tue	Wed	Thu	Fri	Sat
27	28	29	30	31	1	2
3	4	5	6	7	8	9
10	11 **Test 1**	12	13 **Intro**	14	15	16
17	18 **Ch. 04**	19	20 **Ch. 05**	21	22	23
24	25 **Ch. 06**	26	27 **Ch. 07**	28	1	2

March 2019

Sun	Mon	Tue	Wed	Thu	Fri	Sat
24	25	26	27	28	1	2
3	4 **Ch. 08**	5	6 **Ch. 09**	7	8	9
10	11 **Ch. 10**	12	13 **Ch. 11**	14	15	16
17	18	19	20 **Spring Break**	21	22	23
24	25 **Ch. 12**	26	27 **Ch. 13**	28	29	30
31						

April 2019

Sun	Mon	Tue	Wed	Thu	Fri	Sat
31	1 **Test 2**	2	3 **Ch. 14**	4	5	6
7	8 **Ch. 15**	9	10 **Ch. 16**	11	12	13
14	15 **Ch. 17**	16	17 **Ch. 18**	18	19	20
21	22 **Live!!!**	23	24 **Reading**	25	26	27
28	29	30				

Hands-on activities!

- Start Truss Me! on your phone or tablet
- For this activity, please wait for the instructor for each step to avoid spoilers!

Acknowledgements

Thanks!

- Questions?