On the compact wave dynamics of tensegrity metamaterials

metaMAT webinar series

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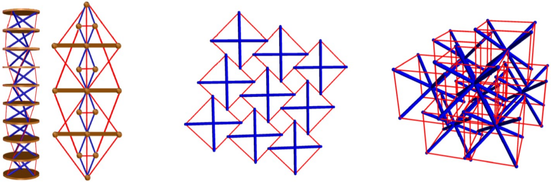
03 March 2021, 02 PM GMT

Outline

Introduction - Tensegrity Metamaterials

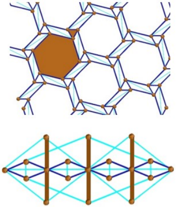
Nonlinear metamaterials are progressively emerging as structured materials that can tune their responses to the level of the applied stress/strain and the amplitude of traveling waves.

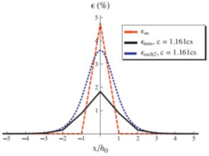
Particularly interesting is the class of tensegrity metamaterials: their mechanical behavior can be effectively adjusted by playing with internal and external prestress, as well as the usual controls of geometry, topology, and properties of the members.



Main goals of ongoing research on tensegrity metamaterials

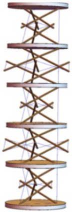
To use fractal shapes to design nature-inspired lattice structures

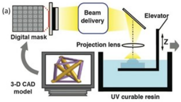


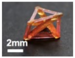


To exploit the highly nonlinear mechanics of tensegrity systems

Unprecedented acoustic applications of tensegrity structures

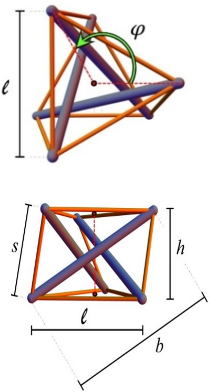






Additive manufacturing (PμSL) with prestress capabilities

1D Preliminaries - Geometrically nonlinear response of tensegrity prisms



Kinematic variables

$\ell,\ h,\ \phi$

Compatibility equations

$$b = \sqrt{h^2 - \frac{2}{3}\ell^2\cos(\phi) + \frac{2\ell^2}{3}}$$

$$s = \frac{\sqrt{3h^2 - \sqrt{3}\ell^2\sin(\phi) + \ell^2\cos(\phi) + 2\ell^2}}{\sqrt{3}}$$

Force densities

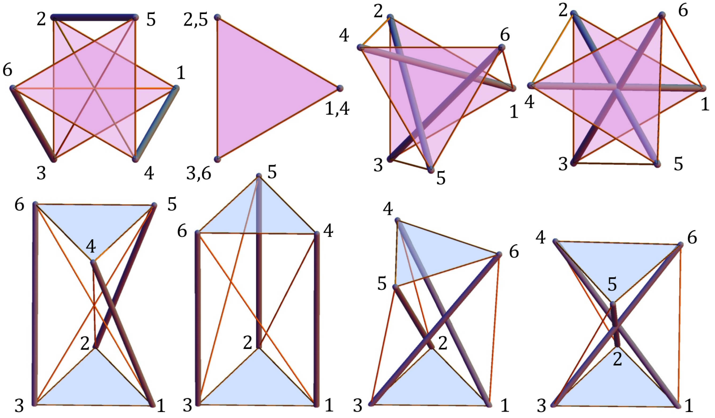
$$x_1 = -\frac{2F\sin(\phi)}{3\sqrt{3}h\left(\sqrt{3}\sin(\phi)+\cos(\phi)\right)}$$

$$x_2 = -\frac{F\left(\sin^2(\phi)-\sqrt{3}\sin(\phi)+\cos^2(\phi)-\cos(\phi)\right)}{9h\left(\sqrt{3}\sin(\phi)+\cos(\phi)\right)}$$

$$x_3 = \frac{F}{3h} - \frac{2F\sin(\phi)}{3\sqrt{3}h\left(\sqrt{3}\sin(\phi)+\cos(\phi)\right)}$$

Work in collaboration with: A. Amendola, G. Carpentieri, M. de Oliveira, R.E. Skelton (Compos Struct, 117, 234-243, 2014; J Mech Phys Solids, 74, 136-157, 2015)

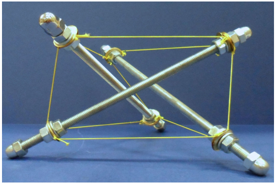
Admissible configurations

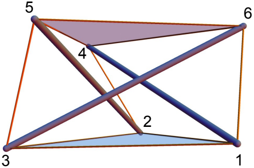


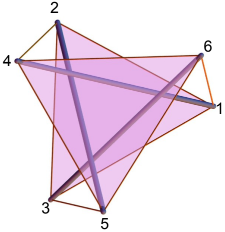
$\varphi = -\frac{\pi}{3}$ $\varphi = 0$ $\varphi = \frac{5}{6}\pi$ $\varphi = \pi$

Fully elastic model

$$x_1 = \frac{1}{s} k_1 (s - s_N), \quad x_2 = \frac{1}{\ell} k_2 (\ell - \ell_N), \quad x_3 = -\frac{1}{b} k_3 (b - b_N)$$



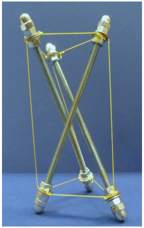


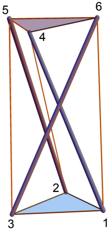


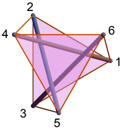
movie ⇑

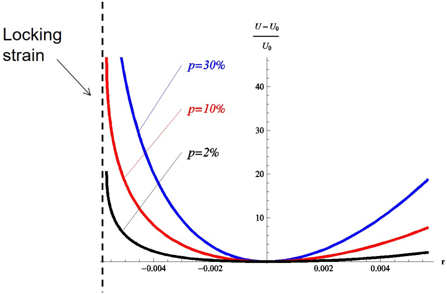
Rigid-elastic model (rigid bases)

$$b = b_0 = \text{const}, \ \ell = \ell_0 = \text{const}$$









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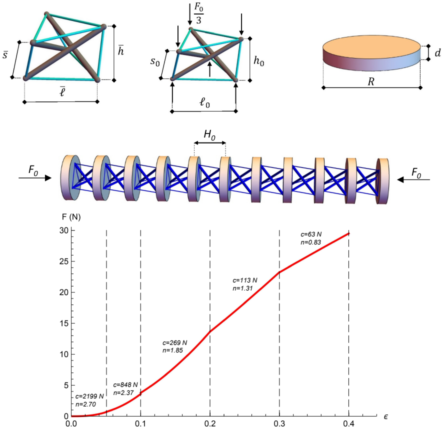
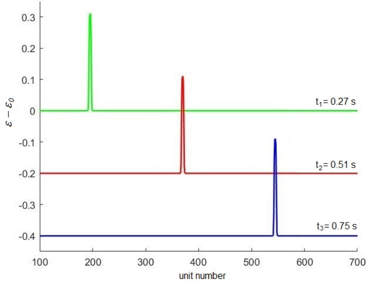
Compact compression waves on 1D tensegrity chains with stiffening response

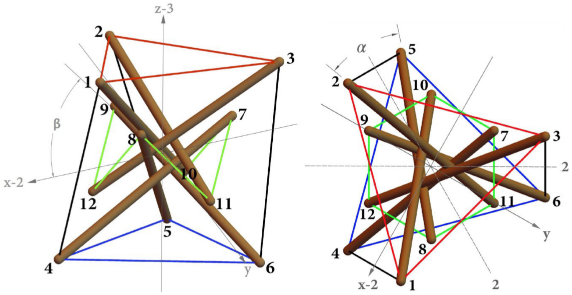




movie ⇑

Work in collaboration with: A. Amendola, G. Carpentieri, C. Daraio, V.F. Nesterenko, L. Senatore, R.E. Skelton (J Mech Phys Solids, 60, 1137-1144, 2012; APL, 105, 201903, 2014)

Class $\theta = 1$ prisms



b = bar length
l = horizontal cables length
c = inner cables length
v = vertical cables length
α = twisting angle between top and bottom bases
β = slope of the internal strings over the horizontal plane

Modano, M., Mascolo, I., Fraternali, F., Bieniek, Z. Numerical and analytical approaches to the self-equilibrium problem of class θ = 1 tensegrity metamaterials. FRONTIERS IN MATERIALS (Mechanics of Materials), 5:5, 2018.

Mascolo, I., Amendola, A., Zuccaro, G., Feo, L., Fraternali, F. On the geometrically nonlinear elastic response of class θ = 1 tensegrity metamaterials. FRONTIERS IN MATERIALS (Mechanics of Materials), 5:16, 2018.

Self-equilibrium problem

Member matrices

$$M = [B \;\; S]$$
$$n_{ik} - n_{jk}$$

Self-equilibrium problem

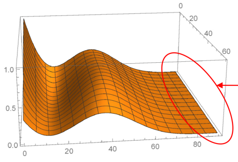
$$Ax = 0$$
$$A = [-B\Lambda \;\; S\Gamma]$$

Λ bars' force densities
Γ strings' force densities

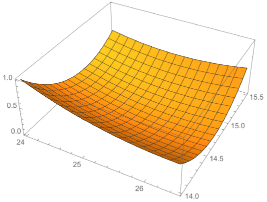
$$A^T A x = 0$$
$$A^T A = G$$

Freestanding configurations

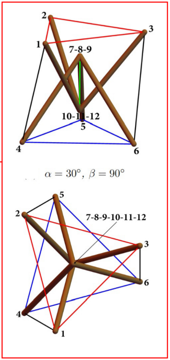
$$\det(G) = f(\alpha, \beta) = 0$$



(a) $\alpha \in [0, 60]° \times \beta \in [0, 90]°$.



(b) $\alpha \in [24.0, 26.5]° \times \beta \in [14.0, 15.5]°$.

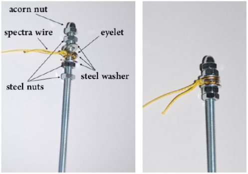


We are interested in the local minumum of f(α,β) that corresponds to a non-degenerate prestressable configuration

Construction of physical models

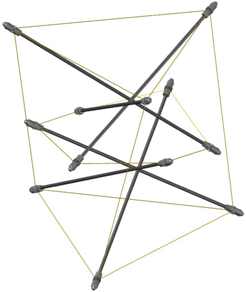
(a) (b) (c) (d) (e)

Connection and securing of Spectra® cables to eyelets and washer to be passed through the threaded bars (a-c); a triangular network of cables (d); a vertical cable (e)



(a) (b)

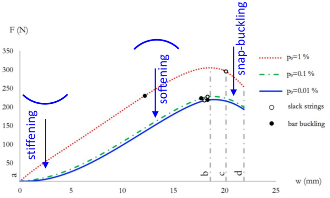
Bar equipment: before (a) and after (b) the fasting of the steel nuts

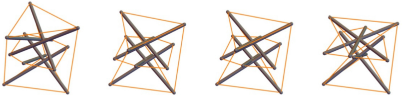


Assembling steps:

(1) mounting of top, bottom, and inner cables on the bars;
(2) insertion of vertical cables;
(3) tightening of the cables.

		S1
E_{bars}	[GPa]	203.53
E_{cables}	[GPa]	30.00
A_{bars}	[mm^2]	8.78
A_{cables}	[mm^2]	0.76
σ_{ybars}	[MPa]	355
$\sigma_{ycables}$	[MPa]	2350



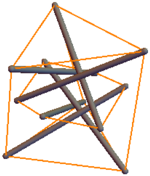


(a)
$F_z = 0\ N$
$u_z = 0\ mm$

(b)
$F_{z_{max}} = 304\ N$
$u_z = 19\ mm$

(c)
$F_z = 295\ N$
$u_z = 20\ mm$

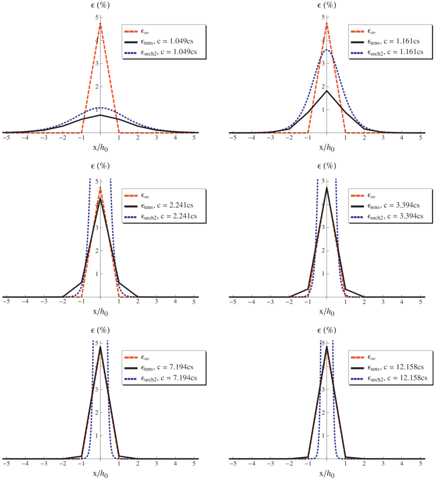
(d)
$F_z = 253\ N$
$u_{z_{max}} = 22\ mm$

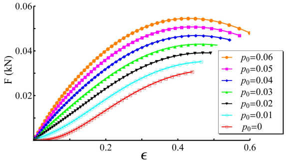


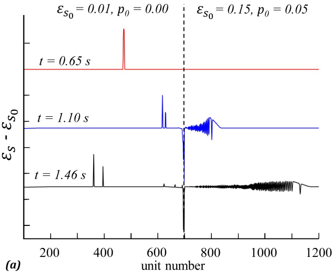
Deformation animation (p=1.0)%

experimental validation ▷

Variation of the wave profile and transition to rarefaction waves







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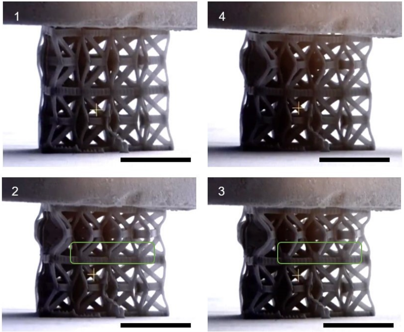
Bistable response of microscale lattices

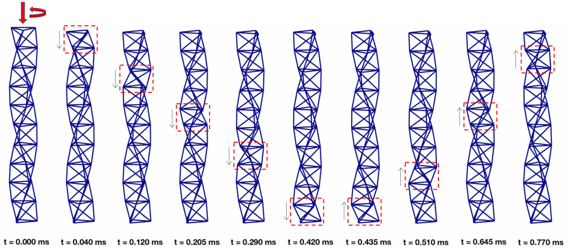


scale bar: 10μm

movie ⇑

Work in collaboration with: C. Grigoropoulos, A. Micheletti, Z. Vangelatos (Nanomaterials, 10, 652, 2020)

Compression solitary waves on a column of bistable tensegrity prisms



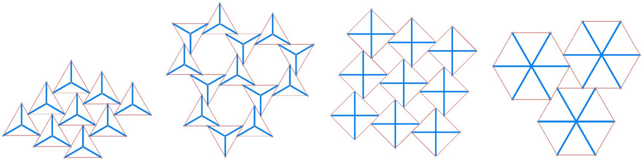
movie ⇑

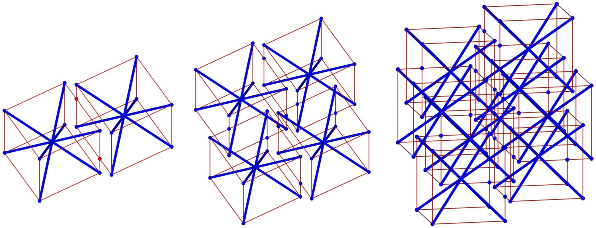
2D and 3D nonlinear tensegrity metamaterials

The secomd part of this talk is focused on the compact wave dynamics in 2D and 3D tensegrity beams and plates with stiffening-type, nonlinear elastic response. The given results in multiple dimensions prove the presence of compact compression waves in tensegrity lattices exhibiting such a behavior.

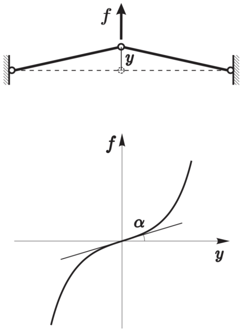
Some distinctive features of 2D and 3D systems are also discussed, which are related to thermalization effects near the impacted zones of the boundary. The analyzed behaviors suggest the use of multidimensional tensegrity lattices for the design and AM of novel sound focusing devices, and novel approaches to Non-Destructive Evaluation (NDE) and Structural Health Monitoring (SHM).

Two- and three-dimensional assemblies



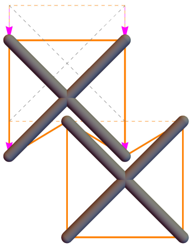


Two-string system with stiffening response



$$f(y) = \frac{2\,k\,(l_0 - \bar{l})}{l_0}\,y + k\,\frac{\bar{l}}{l_0^3}\,y^3 + o(y^5)$$

Two-dimensional stiffening module (interpenetration mechanism)



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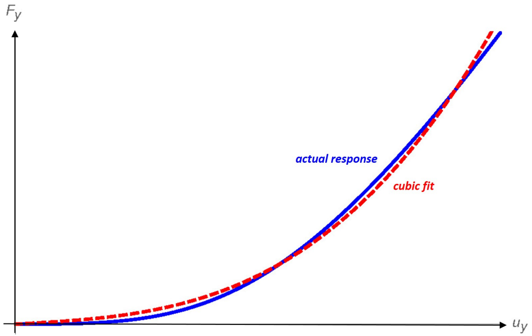
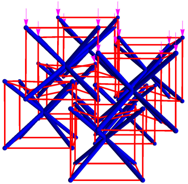
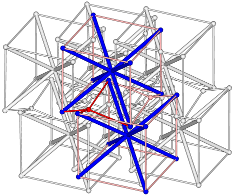
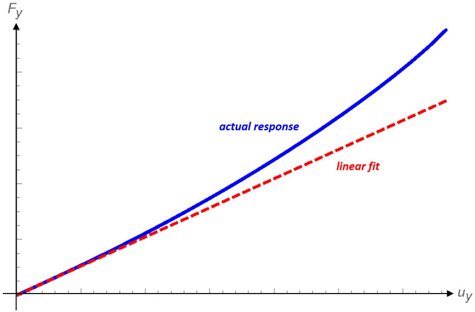


Three-dimensional stiffening module (entaglement mechanism)



movie ⇑





Equations of motion

$$\mathbf{A}(\mathbf{p})\mathbf{t}(\mathbf{p}) + \mathbf{M}\ddot{\mathbf{p}} = \mathbf{0},$$

where $\mathbf{p}$ is the vector of nodal positions, $\mathbf{M}$ is the (constant) mass matrix, $\mathbf{A}(\mathbf{p})$ is the equilibrium operator, and $\mathbf{t}(\mathbf{p})$ is the vector containing the axial forces of all members. The axial force in the i-th member is computed as follows

$$t_i \;=\; \tilde{k}_i(\mathbf{p})\left(l_i(\mathbf{p}) - \bar{l}_i\right),$$

For the cables, we set $\tilde{k}_i(\mathbf{p}) = k_i$ when it results $l_i(\mathbf{p}) \geq \bar{l}_i$, and $\tilde{k}_i(\mathbf{p}) = 0$ when instead it results $l_i(\mathbf{p}) < \bar{l}_i$. The solution $\mathbf{p}(t)$ can be easily obtained through a *Matlab*® script.

Material properties

quantity	value	units
cell side	20	mm
bar diameter	1.75	mm
cable diameter	0.25	mm
Ti6Al4V Young's modulus	120	Gpa
Ti6Al4V mass density	4.42	g/cm^3
Nylon 12 cables Young's modulus	0.5	Gpa
Lead mass density	11.34	g/cm^3
spherical mass radius	2.5	mm
additional nodal mass	0.74	g

Solitary waves on stiffening systems

Solitary wave dynamics of weakly precompressed particulate systems featuring power-law interactions with exponent $n > 1$.

Characteristic phase speed V_s and the spatial length L_n of solitary waves (*Nesterenko, Dynamics of Heterogeneous Materials, Springer, 2001*)

$$V_s = c_n \sqrt{\frac{2}{n+1}} \, \varepsilon_m^{\frac{n-1}{2}}$$

$$L_n = \frac{\pi a}{n-1} \sqrt{\frac{n(n+1)}{6}}$$

Compression solitary waves on 2D beams

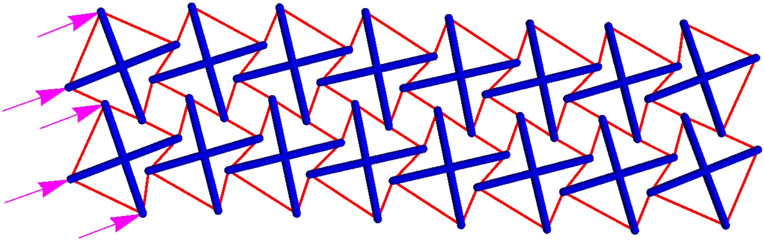
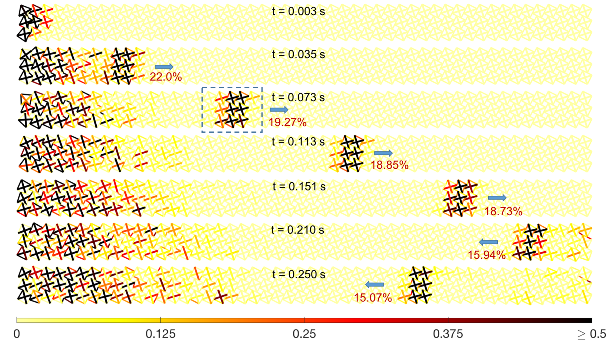


Figure 1: Two-dimensional tensegrity beam under impact loading.

Work in collaboration with: A. Micheletti, G. Ruscica, (Nonlinear Dynam, 98, 2737-2753, 2019)



movie ⇑

Deformed configurations with superimposed energy colormaps for a 3×50 strip impacted with $v_0 = 5\,m/s$.

Effects of different impact velocities

Table 1: Statistics of the wave speed, maximum axial strain of the cables, and total energy fraction of the ccw traveling across Part 2, for different impact velocities.

v_0 (m/s)	$\bar{V}_{ccw}$ (m/s)	$s_{V_{ccw}}$ (m/s)	ε_m (%)	$\delta\hat{E}_{ccw}$ (%)
5.00	5.67	0.02	33.46	1.92
3.75	5.10	0.02	18.41	0.87
2.50	4.27	0.02	8.09	0.65

movie $v_0 = 3.75$ *m/s* ▷

movie $v_0 = 2.50$ *m/s* ▷

We further analyze the partition of the impact energy on examining the time-variation of the energies $E_j(t)$ that are associated with the nodes of Part 1 (thermalized region) and Part 2 (region behind the thermalized region):

$$E_j(t) = \frac{1}{2} m_j v_j^2(t) + \frac{1}{4} \sum_i \tilde{k}_i (l_i(t) - \bar{l}_i)^2$$

Energy correlation function: $C(t, t_0) = c(t)/c(t_0)$, where

$$c(t) = \frac{1}{N_b - N_a} \left\langle \sum_{i=N_a}^{N_b} E_i^2(t) \right\rangle - \left\langle \frac{1}{N_a - N_b} \sum_{i=N_a}^{N_b} E_i(t) \right\rangle^2$$

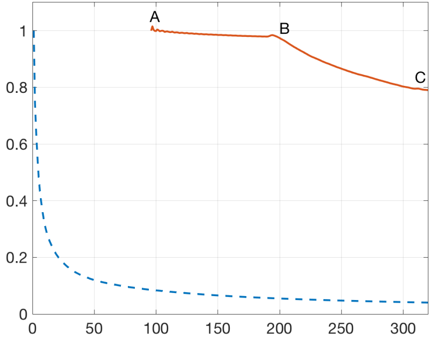
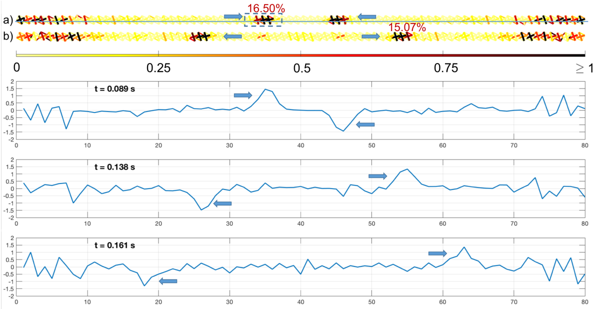


Figure 2: Energy correlation function in Part 1 (dashed blue curve) and Part 2 (solid red curve) for $v_0 = 5$ m/s (times in ms on the x-axis).

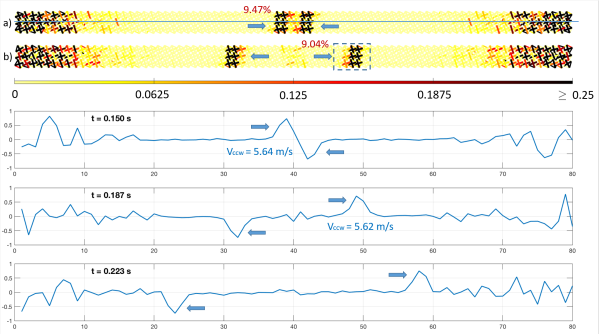
Collision between two ccws on 2D beams - 1/3



movie ⇑

Deformed configurations of a 1×80 strip after double impact with initial velocity $v_0 = 5\,m/s$: a) t=0.089 s; b) t=0.138 s.

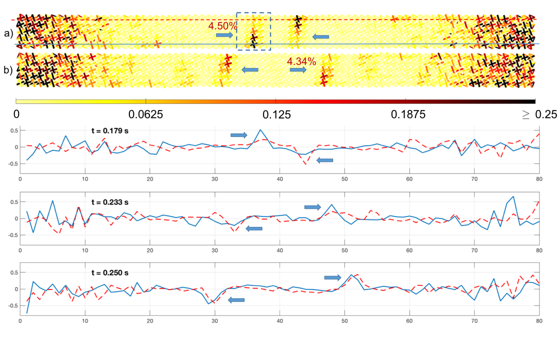
Collision between two ccws on 2D beams - 2/3



movie ⇑

Deformed configurations of a 3×80 strip after double impact with initial velocity $v_0 = 5\,m/s$: a) t=0.150 s; b) t=0.187 s.

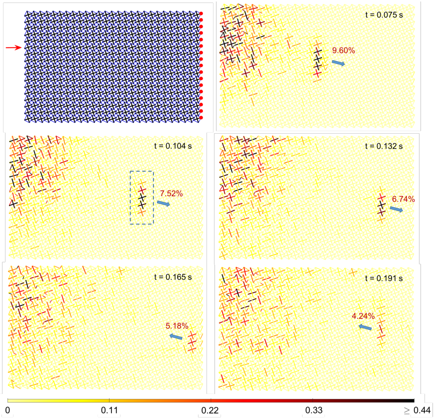
Collision between two ccws on 2D beams - 3/3



movie ⇑

Deformed configurations of a 5×80 strip after double impact with initial velocity $v_0 = 5\,m/s$: a) t=0.179 s; b) t=0.233 s.

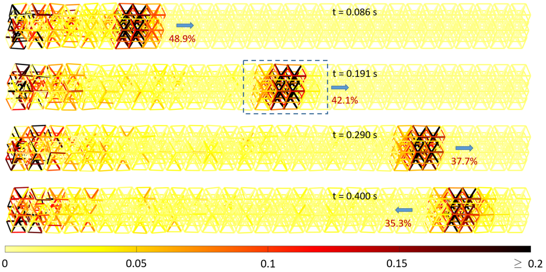
Solitary wave dynamics of a 2D plate



movie ⇑

Work in collaboration with A. Micheletti (to appear)

Compression solitary waves on 3D beams

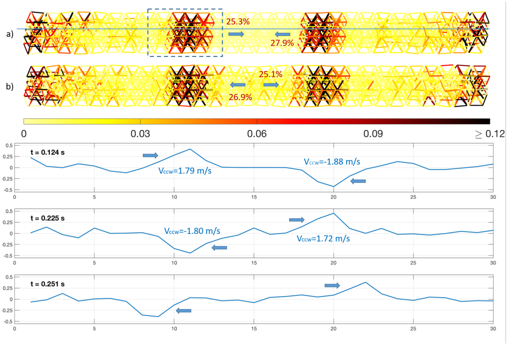


movie ⇑

Deformed configurations and energy colormaps for a $2 \times 2 \times 30$ *beam of cubic cells impacted with* $v_0 = 1.25\, m/s$.

Work in collaboration with: A. Micheletti, G. Ruscica, (Nonlinear Dynam, 98, 2737-2753, 2019)

Collision between two ccws on 3D beams



movie ⇑

Deformed configurations of a $2 \times 2 \times 30$ *beam of cubic cells after double impact with initial velocity* $v_0 = 1.25$ *m/s: a) t=0.124 s; b) t=0.225 s.*

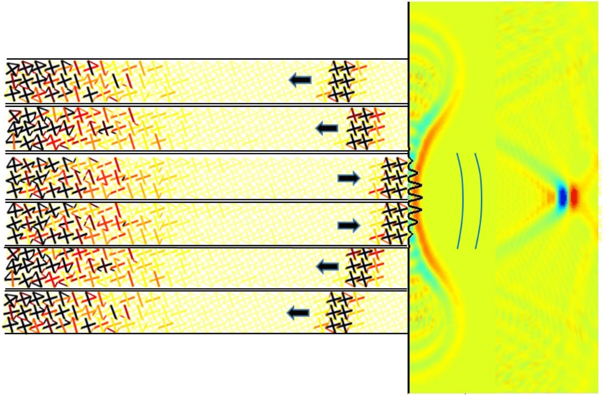


Figure 3: Acoustic lens consisting of an array of tensegrity beams featuring strong hardening response and scalable size of the unit cells.

A parade of numerical simulations of impact events on 2D and 3D tensegrity beams has led us to discover that the impact dynamics of such systems is characterized by the combination of thermalization phenomena in proximity of the impacted areas, and the formation and propagation of compact compression waves in front of the thermalized regions.

The traveling ccws transport energy on localized packets of unit cells spanning from two to three lattice modules in the longitudinal direction. Such compression waves propagate with nearly constant velocity before and after collisions with other ccws, and exhibit limited energy leaking during their propagation

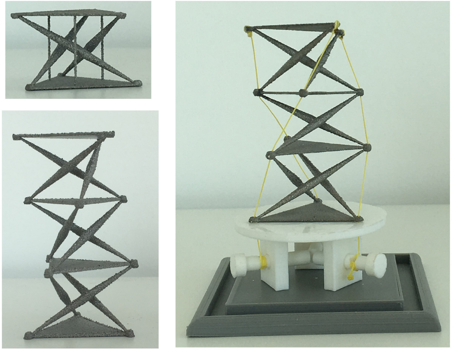
The behaviors examined in the present study suggest the employment of tensegrity beams to form innovative acoustic lenses. Such devices are expected to be able to generate tunable ccws in an adjacent host medium, which will cohalesce at a given focal point (Spadoni and Daraio, PNAS 2010). The latter may consist of a material defect or a tumor mass in the host medium (Daraio and Fraternali, US Pat. No. 8,616,328)

Compared to devices based on granular metamaterials supporting fixed wavelength solitary waves, the tensegrity acoustic lenses will profit from the adjustable width of ccws in tensegrity lattices, and the atomic-scale localization phenomenon observed in the high-energy limit.

Future directions of the present research include:

- multiscale analytic modeling of the wave dynamics of tensegrity lattices
- development of novel additive manufacturing techniques for the fabrication of macro- and micro-scale physical models of lattices
- dynamical tests aimed at experimental validating the puzzling compact wave dynamics of tensegrity metamaterials
- fabrication and testing of acoustic lenses and SHM actuators and sensors with tensegrity architecture

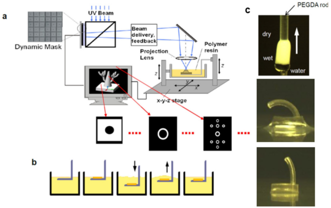
Post-tensioning of 3D printed metallic structures

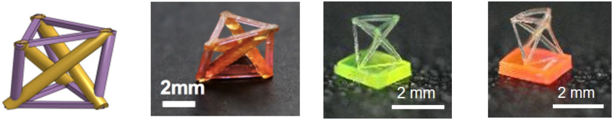


movie ⇑

3D printing of adaptive microscale structures

3D printing of active hydrogels using projection micro-stereolithography: (a) schematic of a PμSL setup and fabrication process; (b) sample substrate movement during fabrication; and (c) water or solvent diffusion in a rod. Commonly employed hydrogels: poly(ethylene glycol) diacrylate (PEGDA); hexanediol diacrylate (HDDA); poly(N-isopropylacrylamide) (PNIPAAm)





Work in collaboration with: H. Lee, C. Grigoropoulos, N. Singh, Z. Vangelatos (to appear)

Thank you for your attention!

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